1. Contact Hours: Lectures will be in Wean Hall 4709 at 12:30pm on Mondays, Wednesdays and Fridays.
   My office hours are
   • Monday, 14:30 - 15:30,
   • Wednesday, 13:20 - 14:30,
   • Friday, 13:20 - 14:30.
   These times are deliberately after lectures (unfortunately I cannot make directly after Mondays lecture). You should also feel free to see me outside office hours if you have questions.

2. Prerequisites: Some prior knowledge of ordinary differential equations is required (either 260 or 261 is sufficient) as well as knowledge from linear algebra (either 240 or 241).

3. Recommended Text: The recommended text is Murray, *Mathematical Biology*, Third Edition, Springer, 2007. There will be a couple of topics that we will cover that will not be in this book, for these I will produce notes. The exercises that will be set for assignments will mostly come from this text.

4. Introduction: Mathematical biology is an extremely diverse and actively growing field which provides excellent opportunity for mathematical modelling. The topics we will study range from large scaling modelling (for example population dynamics) to small scale modelling (for example genetic modelling). The mathematical techniques we will study include Markov chains, branching processes, dynamical systems, differential equations, linear algebra and game theory.

   In general mathematical biology includes the following topics (which is too many to cover in one semester): single and multiple species population modelling, population genetics and evolution, social interaction, chemical reactions, modelling infectious diseases, modelling tumours, pattern formation (why does a tiger have stripes and a leopard have spots?) and many many more. We will study population dynamics, population genetics, molecular biology and modelling infectious diseases. The aim of the course is to give an introduction to these selected topics.

   Whilst we will study problems from mathematical biology the techniques and methods you will learn are more widely applicable. Mathematical modelling approximately follows the method

   ![Modeling Process Diagram]

   Model \(\rightarrow\) Simplify \(\rightarrow\) Analyse \(\rightarrow\) Interpret Output

   Repeat
That is, we start with some phenomena we wish to model and either derive equations based on assumptions that are supposed to reflect reality. Practically these equations will be too complicated to understand directly so we simplify (by for example separating length scales). It is more likely that we will be able to solve these simplified equations or at least perform a ‘better analysis’. However, one should also take care that the approximations made still capture the original phenomena. If not, one should return to the modelling stage and repeat the process. This course will develop these skills which are readily transferable to other sciences/engineering.

The content will be delivered through a combination of lectures and exercise classes. Maths is not a spectator sport and you should be prepared to do questions outside of class to validate your understanding.

5. **Course Assessment:** There will be 8 assignments due approximately every 2 weeks, the 7 best will account for 25% of your overall mark. There will be two midterms, on the 30th September and the 28th October. Each midterm will account for 20% of your final mark. The remaining 35% of your mark will be on the final exam, the date will be confirmed at a later date.

<table>
<thead>
<tr>
<th>Assessment</th>
<th>Date</th>
<th>Contribution to Final Grade</th>
</tr>
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<tbody>
<tr>
<td>8 Assignments</td>
<td>Throughout Semester</td>
<td>7 Best Account for 25%</td>
</tr>
<tr>
<td>Midterm 1</td>
<td>30th Sep</td>
<td>20%</td>
</tr>
<tr>
<td>Midterm 2</td>
<td>28th Oct</td>
<td>20%</td>
</tr>
<tr>
<td>Final</td>
<td>TBC</td>
<td>35%</td>
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The grade boundaries are not fixed but, for your information, will not be higher than 80% for an A, 70% for B, 60% for C, and 50% for a D.

No calculators, other electronic devices, notes or books will be permitted during tests. There will be make up tests for people with legitimate reasons for missing the original test (e.g. documented illness, family emergency, or University sponsored event) if they let me know before the original test. These will be arranged for the following week. No accommodation will be made for students who only let me know after the test apart from in exceptional circumstances.

Deadlines for homework’s will be released at the start of semester and you will always have at least a week from the homework being released to its due date. The due date is a hard deadline. Homework’s received after the deadline without prior approval will receive zero.

In general I will try and be understanding when there is a high work load due to other subjects by, for example, moving deadlines. However I can only do this if I’m made aware and given reasonable notice (at least 3 days). I will be less sympathetic to last minute requests and have no sympathy for requests that come after the deadline.

Some students qualify for special accommodations such as extra time on tests. Please present documentation supporting such a request as soon as possible, and certainly at least one week before the first test. I may not be able to help with last minute requests.

I actively encourage group working however copying assignments is cheating and there will be consequences such as zero marks for the assignment, zero marks for all assignments and failing the course. Don’t do it! This includes copying answers from Google, remember I also have access to the internet. As a rule it is fine to work together to solve homework questions but you should write up on your own.

6. **Content of Course:** We will study the following topics.

**Single Species Population Dynamics** (Chapter 1.1 to 1.5 and 2.1 to 2.5 in Murray). We will look at both discrete models, which take the form of a Markov chain, such as

\[ N_{n+1} = f(N_n) \]

and continuous models, which take the form of a differential equation,

\[ \frac{dN}{dt} = f(N) \]
where $N$ is the number of species in a population. We will be interested in finding steady states, i.e. when does $N_{n+1} = N_n$ or $\frac{dN}{dt} = 0$, and analysing the stability, i.e. if we perturb the steady state $N^*$ by a small number $\Delta N$ then does $N^* + \Delta N$ return to the steady state $N^*$ or to another steady state. The steady states will often depend on parameters, for example birth rate. As we change the parameters we will find that new steady states appear and the stability may change. This gives rise to bifurcations.

**Interacting Species in Population Dynamics** (Chapter 3.1 to 3.5 and 3.9 to 3.10 in Murray). The previous topic only allowed for one species to be modelled at a time. This of course is unrealistic in, for example, predator-prey systems. In this topic we consider systems of (coupled) differential equations:

\[
N_{t+1} = rN_t \exp (aP_t) \\
P_{t+1} = N_t (1 - \exp (-aP_t)).
\]

For the predator-prey systems we let $P$ be the number of predators and $N$ the number of prey. We will also consider the continuous time models:

\[
\frac{dN}{dt} = N(a - bP) \\
\frac{dP}{dt} = P(cN - d).
\]

The questions we ask are analogous to the first topic. We find and characterise the steady states.

**Population Genetics** (I will produce notes for this topic) We model how genes are spread through a population. For simplicity let individuals be genes (think of it as people have one gene which is passed to any offspring without mutation). We make the idealised assumption that the number of individuals $N$ in each generation is fixed and that generation $n+1$ is (mathematically) created by choosing an individual $N$ times from generation $n$. This is an example of a Markov chain and this particular example is called the Wright-Fisher model of population genetics. We will see that some ancestral lines die out whilst others do not. An alternative description is via branching processes. In this model an individual of type $i$ produces an individual of type $j$ with probability $p_{ij}$. We ask what is the expected number of type $j$ individuals in generation $n$. We will also look at whether ancestral lines will die out or grow. This is known as the Galton-Watson process.

**Game Theory** (I will produce notes for this topic). Game theory is the “study of mathematical models of conflict and cooperation between intelligent and rational decision makers”, or in other words how do you choose the ‘best strategy’ when playing a game with rules. We will ask a simple question: Is it better to be a hawk or a dove? Hawks are idealised so that they always fight, this is a high risk strategy compared to a dove which will always choose to run. This strategy means they never make substantial losses (they never get hurt). In a room full of hawks it will be better to be a dove and in a room full of doves it is better to be a hawk. This implies a critical value where it makes no difference. Using game theory we will analyse this model. The techniques you learn in this topic will considerably increase your enjoyment of watching/reading game of thrones.

**Chemical Reactions** (Chapter 6.1 to 6.3 in Murray). Under the assumption that chemicals react proportionally to their concentrations we will derive a system of ODE’s for reactions such as

\[
S + E \xrightleftharpoons[k_{-1}]{k_1} SE \xrightarrow{k_2} P + E
\]

where $S$ is a substrate, $E$ is an enzyme, $P$ is a produce and $k_i$ are parameters controlling the rate of reaction. After deriving the equations we can find the rates of reaction, concentrations of each chemical at any given time and any equilibrium state. We will learn how to non-dimensionalise equations and separate timescales to find approximations that are simpler than the original equations but still capture the right behaviour. The approximations will depend on biological assumptions such as whether it is reasonable to assume that the concentration of the enzyme $E$ is small compared to the substrate $S$, or when there are multiple timescales with one much faster than the other.
Modelling Diseases (Chapter 10.2, 10.3 and 10.10 in Murray). If we let $S$ be susceptibles (individuals that can contract the disease), $I$ infectives (individuals infected with the disease) and $R$ removed (individuals who cannot contract the disease) then one model for the spread of a disease through a population is

$$S \rightarrow I \rightarrow R.$$ 

This is understood as individuals are susceptible until they become infected and after which are removed either through immunity or death. We will study the equations associated with this model and other similar models. Time permitting we will also consider the control (decision making) strategies such as whether it is a good idea to immunise a population.