Complete the following problems. Fully justify each response.

1. How many words \((a_1, a_2, \ldots, a_m) \in [n]^m\) satisfy \(a_1 < a_2 < \cdots < a_m\)?

2. How many integer-valued solutions are there for the following equation, subject to the listed restrictions?
\[
x_1 + x_2 + x_3 + x_4 = 152
\]
(a) \(x_1, x_2, x_3, x_4 > 0\)
(b) \(x_1, x_2, x_3 > 0\) and \(x_4 \geq 0\)
(c) \(x_1, x_2, x_3 > 0\) and \(x_4 \geq 0\) and \(x_2 \leq 15\)

3. Let \(k\) and \(n\) be integers such that \(0 \leq k \leq n - 1\). Provide a combinatorial proof of the identity
\[
\sum_{j=0}^{k} \binom{n}{j} = \sum_{j=0}^{k} \left( \binom{n-1}{k-j} \right) 2^j.
\]
(Note: by a “combinatorial proof,” we mean a proof that is based on counting, rather than an algebraic proof.)