Math 101 Homework

Mary Radcliffe

due 20 Jan 2015

Complete the following problems. Fully justify each response.

1. A graph $G = (V, E)$ is called bipartite if the vertex set can be written as $V = A \cup B$, where $A$ and $B$ are disjoint, and every edge has one endpoint in $A$ and the other in $B$.

   Let $G$ be a graph. Prove that $G$ is 2-chromatic if and only if $G$ is bipartite.

2. Determine the chromatic number of the Petersen graph.

3. A cycle on $n$ vertices (also called an $n$-cycle, denoted $C_n$) is the graph you get by considering an $n$-gon, with the vertices of the $n$-gon the vertices of the graph, and the edges of the $n$-gon the edges of the graph. For example, a 6-cycle and a 7-cycle are shown below.

   Determine $\chi(C_n)$ for any $n$.

Figure 1: $C_6$ (left) and $C_7$ (right)