Discrete Math

Instructor: Mike Picollelli

Day 4
When counting, there are often two simple principles at work:
It’s Combinatorics Time.

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**The Multiplication Principle:** If an event can occur in \( m \) ways, and a second event can occur *independently* in \( n \) ways, then the two events can occur in \( mn \) ways.
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**The Multiplication Principle:** If an event can occur in $m$ ways, and a second event can occur *independently* in $n$ ways, then the two events can occur in $mn$ ways.

**The Addition Principle:** If we can break the objects we are counting into separate, non-overlapping (disjoint) cases, the total number of objects is the sum of the numbers for each individual case.
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But what if we only want the number of permutations of $r$ distinct objects from a collection of $n$?

This number is denoted $P(n, r)$, and, in fact,

$$P(n, r) = \frac{n!}{(n - r)!}.$$
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The number of such permutations is therefore

\[ P(10, 3) = \frac{10!}{(10 - 3)!} = \frac{10!}{7!} = 10 \cdot 9 \cdot 8 = 720. \]
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Later, we will define binomial coefficients, which are written \( \binom{n}{r} \), and show that \( \binom{n}{r} = C(n, r) \). As a result, your instructor, who got very little sleep last night, may end up writing \( \binom{n}{r} \) when he means \( C(n, r) \).
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How many full houses are there in a standard poker deck?
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(Yes, combinatorics *is* all about gambling.)