1 Problems

Putnam 2001/B4. Let $S$ denote the set of rational numbers different from $\{-1, 0, 1\}$. Define $f : S \to S$ by $f(x) = x - 1/x$. Prove or disprove that

$$\bigcap_{n=1}^{\infty} f^{(n)}(S) = \emptyset,$$

where $f^{(n)}$ denotes $f$ composed with itself $n$ times.

Putnam 2001/B5. Let $a$ and $b$ be real numbers in the interval $(0, 1/2)$, and let $g$ be a continuous real-valued function such that $g(g(x)) = ag(x) + bx$ for all real $x$. Prove that $g(x) = cx$ for some constant $c$.

Putnam 2001/B6. Assume that $(a_n)_{n \geq 1}$ is an increasing sequence of positive real numbers such that $\lim a_n/n = 0$. Must there exist infinitely many positive integers $n$ such that $a_{n-i} + a_{n+i} < 2a_n$ for $i = 1, 2, \ldots, n-1$?