1 Problems

Putnam 2002/B4. An integer $n$, unknown to you, has been randomly chosen in the interval $[1, 2002]$ with uniform probability. Your objective is to select $n$ in an odd number of guesses. After each incorrect guess, you are informed whether $n$ is higher or lower, and you must guess an integer on your next turn among the numbers that are still feasibly correct. Show that you have a strategy so that the chance of winning is greater than $2/3$.

Putnam 2002/B5. A palindrome in base $b$ is a positive integer whose base-$b$ digits read the same backwards and forwards; for example, 2002 is a 4-digit palindrome in base 10. Note that 200 is not a palindrome in base 10, but it is the 3-digit palindrome 242 in base 9, and 404 in base 7. Prove that there is an integer which is a 3-digit palindrome in base $b$ for at least 2002 different values of $b$.

Putnam 2002/B6. Let $p$ be a prime number. Prove that the determinant of the matrix
\[
\begin{pmatrix}
x & y & z \\
x^p & y^p & z^p \\
x^{p^2} & y^{p^2} & z^{p^2}
\end{pmatrix}
\]
is congruent modulo $p$ to a product of polynomials of the form $ax + by + cz$, where $a$, $b$, $c$ are integers. (We say two integer polynomials are congruent modulo $p$ if corresponding coefficients are congruent modulo $p$.)