1 Problems

Putnam 1992/B1. Let $S$ be a set of $n$ distinct real numbers. Let $A_S$ be the set of numbers that occur as averages of two distinct elements of $S$. For a given $n \geq 2$, what is the smallest possible number of elements in $A_S$?

Putnam 1992/B2. For nonnegative integers $n$ and $k$, define $Q(n, k)$ to be the coefficient of $x^k$ in the expansion of $(1 + x + x^2 + x^3)^n$. Prove that

$$Q(n, k) = \sum_{j=0}^{k} \binom{n}{j} \binom{n}{k - 2j}.$$  

Putnam 1992/B3. For any pair $(x, y)$ of real numbers, a sequence $(a_n(x, y))_{n \geq 0}$ is defined as follows:

$$a_0(x, y) = x,$$

$$a_{n+1}(x, y) = \frac{(a_n(x, y))^2 + y^2}{2}, \quad \text{for } n \geq 0.$$  

Find the area of the region $\{(x, y) : (a_n(x, y))_{n \geq 0} \text{ converges}\}$. 