1 Problems

Putnam 1992/A1. Prove that \( f(n) = 1 - n \) is the only integer-valued function defined on the integers that satisfies the following conditions.

- \( f(f(n)) = n \), for all integers \( n \);
- \( f(f(n + 2) + 2) = n \), for all integers \( n \);
- \( f(0) = 1 \).

Putnam 1992/A2. Define \( C(\alpha) \) to be the coefficient of \( x^{1992} \) in the power series about \( x = 0 \) of \( (1 + x)^\alpha \). Evaluate

\[
\int_0^1 \left( C(-y - 1) \sum_{k=1}^{1992} \frac{1}{y + k} \right) dy.
\]

Putnam 1992/A3. For a given positive integer \( m \), find all triples \((n, x, y)\) of positive integers, with \( n \) relatively prime to \( m \), which satisfy

\[
(x^2 + y^2)^m = (xy)^n.
\]