1 Problems

**Putnam 1994/A1.** Suppose that a sequence $a_1, a_2, a_3, \ldots$ satisfies $0 < a_n \leq a_{2n} + a_{2n+1}$ for all $n \geq 1$. Prove that the series $\sum_{n=1}^{\infty} a_n$ diverges.

**Putnam 1994/A2.** Let $A$ be the area of the region in the first quadrant bounded by the line $y = \frac{1}{2}x$, the $x$-axis, and the ellipse $\frac{1}{9}x^2 + y^2 = 1$. Find the positive number $m$ such that $A$ is equal to the area of the region in the first quadrant bounded by the line $y = mx$, the $y$-axis, and the ellipse $\frac{1}{9}x^2 + y^2 = 1$.

**Putnam 1994/A3.** Show that if the points of an isosceles right triangle of side length 1 are each colored with one of four colors, then there must be two points of the same color which are at least distance $2 - \sqrt{2}$ apart.