1 Problems

Putnam 1985/A4. Define a sequence \( \{a_i\} \) by \( a_1 = 3 \) and \( a_{i+1} = 3^{a_i} \) for \( i \geq 1 \). Which integers between 00 and 99 inclusive occur as the last two digits in the decimal expansion of infinitely many \( a_i \)?

Putnam 1985/A5. Let \( I_m = \int_0^{2\pi} \cos(x) \cos(2x) \cdots \cos(mx) \, dx \). For which integers \( m \), \( 1 \leq m \leq 10 \) is \( I_m \neq 0 \)?

Putnam 1985/A6. If \( p(x) = a_0 + a_1 x + \cdots + a_m x^m \) is a polynomial with real coefficients \( a_i \), then set
\[
\Gamma(p(x)) = a_0^2 + a_1^2 + \cdots + a_m^2.
\]

Let \( f(x) = 3x^2 + 7x + 2 \). Find, with proof, a polynomial \( g(x) \) with real coefficients such that

(i) \( g(0) = 1 \), and

(ii) \( \Gamma(f(x)^n) = \Gamma(g(x)^n) \)

for every integer \( n \geq 1 \).