1: Given $k$ and a $k$-coloring of a $k$-chromatic graph, prove that for any color $c$ there is a vertex of color $c$ which is adjacent to vertices of every other color.

2, Diestel 5.18: Given a graph $G$ and $k \in \mathbb{N}$ let $P_G(k)$ denote the number of vertex colourings $V(G) \to \{1, \ldots, k\}$. Show that $P_G$ is a polynomial in $k$ of degree $n := |G|$, in which the coefficient of $k^n$ is 1 and the coefficient of $k^{n-1}$ is $-||G||$. ($P_G$ is called the chromatic polynomial of $G$.) (Hint. Apply induction on $||G||$.)

3, Diestel 5.19: Determine the class of all graphs $G$ for which $P_G(k) = k(k - 1)^{n-1}$. (As in the previous exercise, let $n := |G|$, and let $P_G$ denote the chromatic polynomial of $G$.)

Hint: A graph with $n$ vertices is a tree if and only if it is connected and has $n - 1$ edges.