1: For each of chapters 1, 2, 3, 4, 5, 7, 9 and 10, state two major theorems and prove a (not necessarily novel) result using at least one of those theorems.

2: Let $T$ be a tree of order $n$. Let $G$ be a non-empty graph with $\delta(G) \geq n - 1$. Prove from first principles that $T$ is a subgraph of $G$.

3: Prove Grinberg’s Theorem: Let $G$ be a planar Hamiltonian graph with Hamiltonian cycle $C$. Note that $C$ partitions the graph into two polygons: an inner and an outer polygon. Let $f_k$ be the number of $k$-sided polygons inside of $C$ and $g_k$ be the number of $k$-sided polygons outside of $C$. Prove

$$\sum_{k \geq 3} (k - 2)(g_k - f_k) = 0.$$

4: We know from complexity theory that determining whether a graph is Hamiltonian is NP-complete. In other words, there is no known algorithm that runs in polynomial time to determine whether a graph is Hamiltonian, and it is unlikely such an algorithm exists. However, Diestel’s Theorem 10.2.2 gives exact (if-and-only-if) conditions for a degree sequence to be Hamiltonian. How can this be possible?

5: Let $G$ be a graph with $n$ vertices, where $n \geq k + 1$ and $\delta(G) \geq \frac{n+k-2}{2}$. Show that $G$ is $k$-connected.

6, Putnam 2012/B3: A round-robin tournament among $2n$ teams lasted for $2n - 1$ days, as follows. On each day, every team played one game against another team, with one team winning and one team losing in each of the $n$ games. Over the course of the tournament, each team played every other team exactly once. Can one necessarily choose one winning team from each day without choosing any team more than once?