1. Find all solutions to the linear system of equations

\[
x_1 + 2x_2 + 4x_3 = 5 \\
-x_1 + 2x_3 - 2x_4 = 7 \\
-2x_2 - 3x_3 + x_4 = -6.
\]

Express the solution set in vector form.

2. Is the vector \[
\begin{bmatrix}
-4 \\
2 \\
6
\end{bmatrix}
\]
a linear combination of the vectors

\[
\begin{bmatrix}
0 \\
-2 \\
2
\end{bmatrix}, \begin{bmatrix}
2 \\
-2 \\
-2
\end{bmatrix}, \begin{bmatrix}
4 \\
0 \\
-8
\end{bmatrix}.
\]

Either find all such linear combinations, or explain why there are none.

3. \(V\) is a set on which two operations, \(\oplus\) and \(\odot\), are defined. The set is \(\mathbb{R}^2\), and \(\odot\) is the usual scalar multiplication, but \(\oplus\) is defined by

\[
\begin{bmatrix}
u_1 \\
u_2
\end{bmatrix} + \begin{bmatrix}
v_1 \\
v_2
\end{bmatrix} = \begin{bmatrix}
u_2 + v_2 \\
u_1 + v_1
\end{bmatrix}.
\]

(a) Show that vector space axiom 7 is satisfied.

(b) Show that vector space axiom 8 is not satisfied.

(c) Is vector space axiom 3 satisfied? (Naturally, you should provide either a proof or a counterexample.)

4. Let \(P\) represent the statement

\[
\forall x \in \mathbb{R} \ (\exists y \in \mathbb{R} \ (x = y^2 \text{ or } x = -y^2))
\]

(a) Find an expression for the negation of this statement, \(\neg P\). Simplify your expression as much as possible.

(b) One of \(P\) or \(\neg P\) is true. Give a proof for whichever one it is.
Definition 1  Let $V$ be a set with two operations, addition and scalar multiplication. If $u, v \in V$, we denote their sum as $u + v$. If $u \in V$ and $c$ is a scalar, we denote their scalar product as $cu$.

We say that $V$ is a vector space if the following axioms hold for all $u, v, w \in V$ and all scalars $c, d$:

1. $u + v \in V$
2. $u + v = v + u$
3. $(u + v) + w = u + (v + w)$
4. There exists a vector $0 \in V$ such that $u + 0 = u$ for all vectors $u \in V$
5. For each vector $u \in V$ there is a vector $(-u) \in V$ such that $u + (-u) = 0$
6. $cu \in V$
7. $c(u + v) = cu + cv$
8. $(c + d)u = cu + du$
9. $c(du) = (cd)u$
10. $1u = u$