1. In this problem, we will consider the initial value problem
\[ \frac{dy}{dt} = (1 + y^2) \cos(t); \quad y(0) = 0. \]
(a) Find the exact solution (analytically).
(b) Use the MATLAB routine \texttt{eul} to produce an approximate solution on the interval [0, 6], with step size h=1. Determine the maximum error for this approximation by comparing with the exact solution. Repeat this eight times, halving the step size each time. Produce a log-log plot showing maximum error vs. step size.
(c) Repeat part 1b using \texttt{rk2} instead of \texttt{eul}.
(d) Repeat part 1b using \texttt{rk4} instead of \texttt{eul}.
(e) Use \texttt{Matlab} to display on a single diagram the plots of the exact solution and the approximate solutions generated by each of the three methods using a step size of h=.25. Use a distinctive marking for each method. (Note that when you print the graph it will appear in black and white, so simply choosing different colors may not work very well.)

2. A function of the form \( y = ax^b \) is called a power function. Use \texttt{Matlab}'s \texttt{plot} command to sketch the plot of each of the power functions on the interval [0, 2]:
(a) \( y = 2x^3 \)
(b) \( y = 200x^4 \)
(c) \( y = 50x^{-2} \)
Then use the \texttt{loglog} command to produce a log-log plot on the same interval. Describe what the graph of a power function looks like when drawn on a loglog graph.

3. The accuracy of any numerical method in solving a differential equation of the form \( y' = f(t, y) \) depends on how strongly the equation depends on the variable \( y \). (The error bounds depend on the derivatives of \( f \) with respect to \( y \).) To see this experimentally, consider the two initial value problems
\[ y' = y; \quad y(0) = 1 \]
and
\[ y' = e^t; \quad y(0) = 1. \]
Note that \( y(t) = e^t \) is the solution to both problems.
(a) Use \texttt{eul} to compute approximate solutions to the two initial value problems on [0, 1] using a step size of h=.01. Compare the accuracy of the solutions to the two problems.
(b) Repeat this process for the routines \texttt{rk2} and \texttt{rk4}.