§3.1 #20. Differentiate

\( f(t) = \sqrt{t} - \frac{1}{\sqrt{t}}. \)

**Solution:** First, since \( \sqrt{t} = t^{1/2} \), \( f(t) = t^{1/2} - t^{-1/2} \), so by the Difference and Power rules, we have

\[
\frac{d}{dt} (f(t)) = \frac{d}{dt} (t^{1/2} - t^{-1/2}) \\
= \frac{d}{dt} (t^{1/2}) - \frac{d}{dt} (t^{-1/2}) \\
= \frac{1}{2} t^{-1/2} - \left( -\frac{1}{2} \right) t^{-3/2} \\
= \frac{t + 1}{2t\sqrt{t}}.
\]

§3.1 #30. Differentiate \( u = \sqrt{t^2} + 2\sqrt{t^3} \).

**Solution:** First, we rewrite the radicals as powers: \( \sqrt{t^2} = t^{2/3} \) and \( \sqrt{t^3} = t^{3/2} \). Then, we apply the Constant Multiple, Sum, and Power rules:

\[
\frac{d}{dt} (u) = \frac{d}{dt} (t^{2/3} + 2t^{3/2}) \\
= \frac{d}{dt} (t^{2/3}) + \frac{d}{dt} (2t^{3/2}) \\
= \frac{d}{dt} (t^{2/3}) + 2 \frac{d}{dt} (t^{3/2}) \\
= \frac{2}{3} t^{-1/3} + 2 \left( \frac{3}{2} \right) t^{1/2} \\
= \frac{2}{3\sqrt{t}} + 3\sqrt{t}.
\]
§3.1 #46. For what values of $x$ does the graph of $f(x) = x^3 + 3x^2 + x + 3$ have a horizontal tangent?

**Solution:** Horizontal tangents occur wherever $f'(x) = 0$, so we first calculate $f'(x)$:

$$f'(x) = \frac{d}{dx}(x^3 + 3x^2 + x + 3) = \frac{d}{dx}(x^3) + 3 \frac{d}{dx}(x^2) + \frac{d}{dx}(x) + \frac{d}{dx}(3) = 3x^2 + 3(2x) + 1 + 0 = 3x^2 + 6x + 1.$$

Setting $3x^2 + 6x + 1 = 0$, we use the quadratic formula, which gives us

$$x = \frac{-6 \pm \sqrt{6^2 - 4(3)(1)}}{2(3)} = -1 \pm \frac{\sqrt{7}}{3}.$$

Therefore, the graph of $f(x)$ has horizontal tangents at $x = -1 + \frac{\sqrt{7}}{3}$ and $x = -1 - \frac{\sqrt{7}}{3}$.

§3.2 #20. Differentiate

$$y = \frac{\sqrt{x} - 1}{\sqrt{x} + 1}.$$

**Solution:** Since $y$ is clearly a quotient, we’ll attempt to apply the Quotient Rule: let $f(x) = \sqrt{x} - 1$ and $g(x) = \sqrt{x} + 1$, so

$$y = \frac{f(x)}{g(x)}.$$

Then $f'(x) = (1/2)x^{-1/2} = 1/(2\sqrt{x})$ and $g'(x) = (1/2)x^{-1/2} = 1/(2\sqrt{x})$ by the Power and Sum/Difference rules. Therefore, by the Quotient Rule,

$$\frac{d}{dx}(y) = \frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2} = \frac{(\sqrt{x} + 1)(1/(2\sqrt{x})) - (\sqrt{x} - 1)(1/(2\sqrt{x}))}{(\sqrt{x} + 1)^2} = \frac{1}{\sqrt{x}(\sqrt{x} + 1)^2}. $$
§3.2 #24. Find an equation to the tangent line of

\[ y = \frac{\sqrt{x}}{x + 1} \]

at the point (4, 0.4).

**Solution:** Since we already have the point, we need to find the slope of the tangent line, which we know is the derivative at \( x = 4 \). Since \( y \) is a quotient, we’ll have to apply the Quotient Rule, and we first note that \( \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}} \) and \( \frac{d}{dx}(x + 1) = 1 + 0 = 1 \), so

\[
\frac{dy}{dx} = \frac{d}{dx} \left( \frac{\sqrt{x}}{x + 1} \right) = \frac{(x + 1)(1/(2\sqrt{x})) - (\sqrt{x})(1)}{(x + 1)^2} \quad \text{(Quotient Rule)}
\]

\[
= \frac{1/(2\sqrt{x}) - \sqrt{x}/2}{(x + 1)^2}
\]

\[
= \frac{1 - x}{2\sqrt{x}(x + 1)^2}.
\]

Therefore the slope of the line is

\[
\left. \frac{dy}{dx} \right|_{x=4} = \frac{1 - 4}{2\sqrt{4}(4 + 1)^2} = -\frac{3}{100}.
\]

Hence, by the point-slope form, the tangent line is

\[ f(x) = -\frac{3}{100}(x - 4) + 0.4. \]

§3.2 #32. If \( f(3) = 4, g(3) = 2, f'(3) = -6 \), and \( g'(3) = 5 \), find the following numbers:

(a) \( (f + g)'(3) \).

**Solution:** By the Sum Rule, \( (f + g)'(3) = f'(3) + g'(3) = -6 + 5 = -1 \).

(b) \( (fg)'(3) \).

**Solution:** By the Product Rule,

\[
(fg)'(3) = f(3)g'(3) + g(3)f'(3) = (4)(5) + (2)(-6) = 8.
\]
(c) \((f/g)'(3)\).

**Solution:** By the Quotient Rule,
\[\left(\frac{f}{g}\right)'(3) = \frac{g(3)f'(3) - f(3)g'(3)}{g(3)^2} = \frac{(2)(-6) - (4)(5)}{(2)^2} = -8.\]

(d) \((f - g)'(3)\).

**Solution:** By the Quotient Rule,
\[\left(\frac{f - g}{f - g}\right)'(3) = \frac{(f(3) - g(3))f'(3) - f(3)(f'(3) - g'(3))}{(f(3) - g(3))^2} = \frac{(4 - 2)(-6) - (4)(-6 - 5)}{(4 - 2)^2} = 8.\]

§3.2 #42. Find the equations of the tangent lines to the curve
\[f(x) = \frac{x - 1}{x + 1}\]
that are parallel to the line \(x - 2y = 2\).

**Solution:** First, we write the line \(x - 2y = 2\) as \(y = (1/2)x - 1\). The tangent lines parallel to this line have slope \(1/2\), so we find the values of \(x\) where the derivative of \((x - 1)/(x + 1)\) is \(1/2\):
\[\frac{1}{2} = \frac{d}{dx} \left(\frac{x - 1}{x + 1}\right) = \frac{(x + 1)(1) - (x - 1)(1)}{(x + 1)^2} = \frac{2}{(x + 1)^2}.\]

Solving, we get that \((x + 1)^2 = 4\), and therefore \(x = -1 \pm 2\).

For \(x = -1 - 2 = -3\) : \(f(x) = f(-3) = 2\), so the tangent line is \(y = (1/2)(x + 3) + 2 = (1/2)x + 7/2\).

For \(x = -1 + 2 = 1\) : \(f(x) = f(1) = 0\), and the tangent line is \(y = (1/2)(x - 1) = (1/2)x - 1/2\).