1. A cloth manufacturer sells rolls of cloth in $n$ widths $\ell_1, \ell_2, \ldots, \ell_n$. Production is only in widths of width $L$. The manufacturer has to meet demand for $d_j$ rolls of width $\ell_j$ and these must be cut from the larger rolls. For example if $\ell_1 = 7$ and $\ell_2 = 5$ and $L = 36$ then the manufacturer can cut 4 rolls of width 7 and 1 roll of width 5 from one large roll, leaving 3 feet of waste.

The manufacturer wishes to meet demand and minimise total waste. Write an Integer Programming Formulation for this problem. The manufacturer will have to cut up several rolls in several different ways to solve this problem.

2. Solve the following problem by a cutting plane algorithm:

$$\text{minimise } 4x_1 + 5x_2 + 3x_3$$

subject to

$$2x_1 + x_2 - x_3 \geq 2$$
$$x_1 + 4x_2 + x_3 \geq 13$$

$$x_1, x_2, x_3 \geq 0 \text{ and integer.}$$

3. Solve the following problem by a branch and bound algorithm:

$$\text{Maximise } x_1 + 2x_2$$

subject to

$$2x_1 + x_2 \leq 7$$
$$-x_1 + x_2 \leq 3$$

$$x_1, x_2 \geq 0.$$