1. In a take-away game, the set $S$ of the possible numbers of chips to remove is finite. Show that the Grundy numbers $g$ satisfy $g(n) \leq |S|$ where $n$ is the number of chips remaining.

2. Consider the following take-away game: In the first move you are not allowed to take the whole pile. After that, if a player removes $x$ chips, then the next player can remove up to $\lfloor 5x/4 \rfloor$ chips. Determine the $P$ positions.

3. Find the set of $P$-positions for the take-away games with subtraction sets

   (a) $S = \{1, 3, 7\}$.

   (b) $S = \{1, 4, 6\}$.

   Suppose now that there are two piles and the rules for each pile are as above. Now find the $P$ positions for the two pile game.