Mappings

# mappings from $[n] \rightarrow [m] = m^n$

Choose mapping $f$.

$f(1) \quad f(2) \quad \ldots \quad f(n)$

$(m \text{ choices}) \times (m \text{ choices}) \times \ldots \times (m \text{ choices}) = m^n$
\[ \psi(n) = \# \text{Subsets of } [n] \]
\[ = 2^n \]
# of odd cardinality subset of \([n]\)

= \# of even cardinality " "

= \(2^{n-1}\)

\(n\) is odd: \(5\) \(5\)

odd even \(\bar{S} = \exists x \neq S\)

\(n\) even?
$f(A) = \begin{cases} A & \text{if } A \text{ is odd} \\ \{A \cup \{n\} \mid A \text{ is even} \end{cases}$

All subsets of $\{n - 1\}$

Odd subsets of $\{n\}$
\( m^n \) functions from \([n]\rightarrow[n] \) to \([m]\) with permutations of \([n]\)

How many are 1-1?

\[
f(1), f(2), f(3)
\]

(\( m \) choices) x (\( m-1 \) choices) x (\( m-2 \) choices)

How many are onto?

= \( m(m-1)...(m-n+1) \)

= \# sequence \( a_1, a_2, ..., a_n \)

where \( a_i \neq a_j \)

\( i \neq j \)

Inclusion-Exclusion
Binomial Coefficients

$X$ is a finite set.

$$\binom{X}{k} = \frac{1}{k!} \text{ subsets of } X \text{ of size } k$$

$$\left| \binom{X}{k} \right| = \binom{1 \times 1}{k} \quad \text{where} \quad \binom{n}{k} = \frac{n(n-1) \ldots (n-k+1)}{k!}$$

$$\ldots \equiv \text{size}$$
\[
\left| \binom{X}{k} \right| k! = 1 \times (1 \times (1 - 1)) (1 \times (1 - 2)) \ldots (1 \times (1 - k+1))
\]

Choose a set of size \( k \) in any order and delete the \( k \) elements.

\[= \text{ choose a sequence length } k, \text{ with distinct items.} \]