1. Let $\mathcal{A}$ be an intersecting family of subsets of $[n]$ such that $A \in \mathcal{A}$ implies $k \leq |A| \leq \ell \leq n/2$. Show that

$$|\mathcal{A}| \leq \sum_{i=k}^{\ell} \binom{n-1}{i-1}.$$ 

2. Let $m = \lfloor n/2 \rfloor$. Describe a family $\mathcal{A}$ of size $2^{n-1} + \binom{n-1}{m-1}$ that has the following property: If $A_1, A_2 \in \mathcal{A}$ are disjoint then $A_1 \cup A_2 = [n]$. 

3. Consider the following game: There is a pile of $n$ chips. A move consists of removing any proper factor of $n$ chips from the pile. (For the purposes of this question, a proper factor of $n$, is any factor, including 1, that is strictly less than $n$). The player to leave a pile with one chip wins. Determine the $N$ and $P$ positions and a winning strategy from an $N$ position.