1. Prove that for any $k, n \geq 1$ that
\[
\sum_{a_1 + \cdots + a_{2k} = n \atop a_1, \ldots, a_{2k} \geq 0} (-1)^{a_1 + \cdots + a_k} \binom{n}{a_1, \ldots, a_{2k}} = 0.
\]

2. (a) Let $\mathcal{S}_k$ denote the collection of $k$-sets $\{1 \leq i_1 < i_2 < \cdots < i_k \leq m - 3\} \subseteq [m]$ such that $i_{t+1} - i_t \geq 4$ for $1 \leq t < k$. Show that
\[
|\mathcal{S}_k| = \binom{m - 3k}{k}.
\]

(b) How many of the $4^n$ sequences $x_1x_2 \cdots x_n$, $x_i \in \{a, b, c, d\}$, $i = 1, 2, \ldots, n$ are there such that $abcd$ does not appear as a consecutive subsequence e.g. if $n = 6$ then we include $adbbcc$ in the count, but we exclude $aabcda$.

[You should use Inclusion-Exclusion and expect to have your answer as a sum.]

3. How many ways are there of placing $m$ distinguishable balls into $n$ boxes so that no box contains more than $m/2$ balls.
(You should use Inclusion-Exclusion and expect to have your answer as a sum.)