1. Let $\chi(G)$ be the chromatic number of graph $G = (V, E)$. Let $\alpha(G), \kappa(G)$ denote the size of the largest independent set of $G$, clique of $G$ respectively.

Show that

$$\chi(G) \geq \max \left\{ \frac{|V|}{\alpha(G)}, \kappa(G) \right\}.$$ 

Show further that $\chi(G)\chi(\bar{G}) \geq |V|$. Here $\bar{G}$ is the complement of $G$.

2. Given any sequence of $n$ integers, positive or negative, not necessarily all different, show that some consecutive subsequence has the property that the sum of the members of the subsequence is a multiple of $n$.

3. Suppose that $a_1, a_2, \ldots, a_n \in [n]$ and $b_1, b_2, \ldots, b_n \in [n]$. An interval $I$ is a set of the form $\{i, i + 1, \ldots, j\}$. Let $a_I = a_i + a_{i+1} + \cdots + a_j$ and $b_I = b_i + b_{i+1} + \cdots + b_j$. Show that there exist intervals $I, J$ such that $a_I = b_J$. 