1. Using Cayley’s formula, show that the graph obtained from $K_n$ by deleting one edge has exactly $(n - 2)n^{n-3}$ spanning trees.

2. Let $G = (V, E)$ be an $r$-regular graph with $n$ vertices i.e. every vertex has degree $r$. $S \subseteq V$ is a dominating set if $w \notin S$ implies that there exists $v \in S$ for which $\{v, w\} \in E$. Show, by the probabilistic method, that $G$ has a dominating set of size at most $\frac{1 + \ln r}{r} n$.

3. Let $G = (V, E)$ be a graph with minimum degree at least three. Show that it contains a cycle of even length. (Hint: Consider a longest path).