1. Show that the number of sequences out of \( \{a, b, c\}^n \) which do not contain a consecutive sub-sequence of the form \( xx \) where \( x = a, b \) satisfies the recurrence \( b_0 = 1, b_1 = 3 \) and

\[
b_n = b_{n-1} + 2(b_{n-2} + \cdots + b_0) + 2b_0.
\]

[Hint: Consider the number of sequences where the first \( c \) from the left is at position \( k \).]

Deduce from this that

\[
b_n = 2b_{n-1} + b_{n-2}.
\]

2. Show that the number of sequences out of \( \{a, b, c\}^n \) which do not contain a consecutive sub-sequence of the form \( abc \) satisfies the recurrence \( b_0 = 1, b_1 = 3, b_2 = 9 \) and

\[
\begin{align*}
b_n &= 2b_{n-1} + c_n \quad (1) \\
c_n &= c_{n-1} + b_{n-2} + c_{n-2} + b_{n-3} \quad (2)
\end{align*}
\]

where \( c_n \) is the number of such sequences that start with \( a \).

Now find a recurrence only involving \( b_n \), by using (1) to eliminate \( c_n \) from (2).

3. Let \( a_0, a_1, a_2, \ldots \) be the sequence defined by the recurrence relation

\[
a_n + 3a_{n-1} + 2a_{n-2} = n \quad \text{for } n \geq 2
\]

with initial conditions \( a_0 = 1 \) and \( a_1 = 3 \). Determine the generating function for this sequence, and use the generating function to determine \( a_n \) for all \( n \).