1. Use induction to show that
\[
\binom{n-1}{k} = \binom{n}{k} - \binom{n}{k-1} + \cdots \pm \binom{n}{0}.
\]

2. (a) Let \( S_k \) denote the collection of \( k \)-sets \( \{1 \leq i_1 < i_2 < \cdots < i_k \leq m - 2\} \subseteq [m] \) such that \( i_{t+1} - i_t \geq 3 \) for \( 1 \leq t < k \). Show that
\[
|S_k| = \binom{m-2k}{k}.
\]

(b) How many of the \( 3^n \) sequences \( x_1x_2 \cdots x_n, \ x_i \in \{a, b, c\}, \ i = 1, 2, \ldots, n \) are there such that \( abc \) does not appear as a consecutive subsequence e.g. if \( n = 6 \) then we include \( aabbcc \) in the count, but we exclude \( aabcba \).

[You should use Inclusion-Exclusion and expect to have your answer as a sum.]

3. Find an expression for the size of the set
\[
\{(x_1, x_2, \ldots, x_m) \in Z^m : x_1 + x_2 + \cdots + x_m = n \text{ and } a \leq x_j \leq b \text{ for } j = 1, 2, \ldots, m\}.
\]

[You should use Inclusion-Exclusion and expect to have your answer as a sum.]