$9/17/2008$

$A_n = \# \text{ of objects of some type.}$

We want to know what $A_n = \ldots$

Unknown sequence $a_0, a_1, a_2, \ldots, a_n, \ldots$

Sometimes we can find equation $A_n = \text{function}(a_{n-1}, a_{n-2}, \ldots)$

and solve.
$b_n =$ number of sequences in $\{a,b,c\}^n$

that do not contain aa as a consecutive subsequence.

$b_1 = 3$

$b_2 = 8$

$b_n = 2b_{n-1} + 2b_{n-2}$
$b_n = 2b_{n-1} + 2b_{n-2}$
A has \( n \) to spend. Each day, buy \( B \) \( \$1 \) Buns, Ice-Cream or Pastry.

Purchase: B B P I I P B

How many ways to spend money?

\[ U_n = U_{n-1} + 2U_{n-2} \]

\[ u_0 = u_1 = 1 \]
Unknown sequence
\[ a_0, a_1, a_2, \ldots, a_n, \ldots \]
equivalent to

Unknown function
\[ a(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n + \ldots \]

Recurrence \( a_0, a_1, \ldots, a_n, \ldots \)
leads to
Equation for \( A(x) \)
\[(1 + x)^\alpha = \sum_{n=0}^{\infty} \binom{\alpha}{n} x^n\]

True for arbitrary \(\alpha\)

\[\binom{\sqrt{2}}{4} \cdot \frac{\sqrt{2} (\sqrt{2} - 1)(\sqrt{2} - 2)(\sqrt{2} - 3)}{4!}\]
\[ u_n - 5u_{n-1} + 6u_{n-2} = 0 \quad n \geq 2 \]

\[ u_0 = 1, \quad u_1 = 3 \quad \Rightarrow \quad u(n) = u_0x^n + u_1x^{n-1} + \ldots + u_nx^n + \ldots \]

\[
\sum_{n=2}^{\infty} (u_n - 5u_{n-1} + 6u_{n-2})x^n = 0, \quad \forall x \]

\[
\sum_{n=2}^{\infty} u_n x^n - 5 \sum_{n=2}^{\infty} u_{n-1} x^n + 6 \sum_{n=2}^{\infty} u_{n-2} x^n = 0
\]

\[
\tag{\text{u}(x) - u_0 - u_1 x} \quad \tag{\text{x} (\text{u}(x) - u_0)} \quad \tag{\text{x}^2 \text{u}(x)}
\]

\[
\sum_{n=2}^{\infty} u_{n-1} x^{n-1} = u_1 x + u_2 x^2 + \ldots
\]
\[
\sum_{n=2}^{\infty} u_n x^n - 5 \sum_{n=2}^{\infty} u_{n-1} x^n + 6 \sum_{n=2}^{\infty} u_{n-2} x^n = 0
\]

\[u(x) - u_0 - u_1 x - xc(u(x) - u_0) - x^2 u(x)\]

\[
\sum_{n=2}^{\infty} u_{n-1} x^{n-1} = u_1 x + u_2 x^2 + \ldots 
\]

\[u(x) - 1 - 3x - 5xc(u(x) - 1) + 6x^2 u(x) = 0\]

\[u(x) \left[ 1 - 5x + 6x^2 \right] = \frac{1 + 3x - 5x}{1 - 2x} = \frac{1 - 2x}{1 - 5x + 6x^2}\]

\[u(x) = \frac{1 - 2x}{1 - 5x + 6x^2}\]
\[ U_n = 3^n. \]