$\mathcal{V}_{x, G} = \frac{1}{|G|} \sum_{g \in G} |F_{ux}(g)|$

$n = 2m$

2 colors
\[ V_{x,G} = \frac{1}{|G|} \sum_{g \in G} |F_{uv}(g)| \]

\[ n = 2m \]

\[ b \]

\[ c \]

\[ d \]

\[ e \]

\[ f \]

\[ g \]

\[ h \]

\[ i \]

\[ j \]

\[ k \]

\[ l \]

\[ m \]

\[ n \]

\[ o \]

\[ p \]

\[ q \]

\[ r \]

\[ s \]
\[ P \leq \mathcal{A} \]

\[ n \leq 2n \]

\[ g \leq e \]

\[ \text{colors} \]

\[ \frac{1}{\text{Fix}(\mathcal{A})} \]

\[ 2^{n^2} \]

\[ 2^{n^2/4} \]

\[ b \]

\[ c \]

\[ 2 \]

\[ 2 \]

\[ 2 \]

\[ 2 \]

\[ 2 \]

\[ 2 \]

\[ 2 \]

\[ 2 \]

\[ 2 \]

\[ n/(n+1)/2 \]

\[ 2 \]
$g$ is partitioning $D$ into cycles. All things in same cycle have same color.
Cycles of a permutation

\[ \Pi : D \rightarrow D \]

\[ \Gamma \] in a digraph

\[ \Pi \]

Example

\[ \begin{array}{ccccccccccc}
   i & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
   \pi(i) & 6 & 3 & 1 & 0 & 5 & 1 & 9 & 7 & 2 & 4 & 8 \\
\end{array} \]

Union of disjoint cycles
Polya's Theorem

Domain $X = \{ x : D \rightarrow C \}$

Colors

Group $G = \{ \text{permutations of } D \}$

"Induce" permutation on $X$

$g \in X$

$g \times n = ?$

$\forall c \in X, g \times n \in X$

$g \times x (d) = x(g^{-1}(d))$

$g$

color of cl under $g \times x$
$D + \text{coloring} = \infty$

\[ g : D \rightarrow D \]

New coloring \[ g \ast \pi \]

\[ g \ast \pi (d) = \pi (g^{'} (d)) \]
Goal

We know how to count “distinct” colorings of $X$

Suppose we have 2 colors, R & B

$|O| = 100$

How many “distinct” colorings have 60 R & 40 B.
Pattern Inventory

\[ x \in X \]

Colors are \( R, B, G \).

\[ x(d) \]

\[ \omega_{x(d)} \equiv \text{"weight" of a color}. \]

If \( C = R, B, G, \ldots \),

\[ w_R = R, \quad w_B = B, \quad \omega (x) = \prod_d \omega_{x(d)} \]

\[ R^3 B \]
\[ P I = \sum_{c \in S^*} \omega(c) \]

S* contains one coloring from each orbit.

Coefficient of \( R^3 B^3 \)

\( \text{ord}_4 = \# \text{ of distinct colorings with } 3R \& 3B \)
Ex: 2

\[ P \Sigma = R^4 + R^3 B + 2R^2 B^2 + RB^3 + B^4 \]