m balls (distinguishable)
n boxes [≡ colors]

\[ Z = \# \text{ non-empty boxes} \]
\[ = Z_1 + Z_2 + \cdots + Z_n \]
\[ Z_i = 1, \text{ non-empty} \]
\[ E(Z) = n \cdot E(Z_1) \]
\[ = n \cdot \Pr(Z_1 > 0) \]

\[
\Pr(\text{Box 1 is non-empty})
\]
\[ = 1 - \Pr(\text{Box 1 is empty}) \]
\[ = 1 - \frac{(n-1)^m}{n^m} \]
Distinguishable vs Indistinguishable

3 balls
2 colors

Distinguishable

RRR
RRB
RRB
RRB
RRB

Indistinguishable

3R 0B
2R 1B
1R 2B
1R 2B
3B
There are $n$ people with hats on their heads. Hat colors have been randomly colored Red/Blue.
Problem:
Each Person can say
(1) My hat is Red
(2) My hat is Blue
(3) I don't know

If someone guess their hat Color
and no one guesses wrong Color
BIG WIN
BIG LOSS if guess wrong color.
Claim: \exists \text{ strategy s.t.}

\[ P_r(\text{ winning }) = 1 - o(1) \]

\[ Q_n = \mathbb{Z}_0, 1 \mathbb{Z}^n \] — stands for \( R \) \( B \) hat colors

A coloring is an element of \( Q_n \)

\[ h(x, y) = \begin{cases} 1 & \text{if } y \in L \cup W \\ 0 & \text{otherwise} \end{cases} \]

\[ h(x, y) = \# \{ \nu : x \neq y, \nu \in \mathbb{N} \} \]
Every one "assurance" hold caring

"Fixed" LM - L.E. very
people and room "Sharing"
Is there a small $L$?

Let $p = \frac{\text{ln } n}{n}$.

Choose $L^2$ randomly by placing points into $L^2$ with probability $p$.

Choose $L$ uniformly from $L^2$.
\[ E(1L_1 \cup L_2) \]
\[ = E(1L_1) + E(1L_2) \]
\[ \leq 2^n p + 2^n e^{-np} \leq 2^n \cdot \frac{a \log n}{n^b} \]
\[ 1-p \leq e^{-p} \]
\[ E(\Omega) \leq 2^n \times \frac{2 \log n}{n} \]

\[ \Rightarrow \exists \text{ cover } G \text{ size } \leq 2^n \times \frac{2 \log n}{n} \]

\[ \text{ otherwise } \quad E(\Omega) > 2^n \times \frac{2 \log n}{n} \]
Union Distinct Families

\[ A \text{ is a collection of subsets } \mathcal{G} \subseteq [n] \]

\[ A \text{ is union distinct if } A \cup B \neq C \cup D \text{ for distinct } A,B,C,D \]
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