Claim: If \( n \geq N(k, k, 3) \), then \( \exists \|Y\| \)

\( (i) \) \( \|Y\| = k \)

\( (ii) \) \( \text{Y are nearby} \)

\( (iii) \) \( \text{Convex} \)

\( \text{Not} \)
If every 4-subset of $Y \subseteq X$ forms convex quadrilateral, then $Y$ forms convex polygon.

Vertices of $X$ convex hull.

$Y$ is convex iff

$Y = $ convex hull.

$Y$ is not convex.

Find

Triangulate Convex hull.
If $|X| = n \geq N(k, k; 3)$, then for all tuples from $X$, $|Y| = k$.

Take 3 points $X_i, X_j, X_k$ with $i < j < k$. Each point is assigned a color:

- **Red** for $X_i$
- **Blue** for $X_j$
- **Blue** for $X_k$
If a k-set will all triples the same color:

We have to show

All 4 triples have same color.
Assume \( a < b < c \)

(i) \( a < b < c \)

(ii) \( a < b < d \)
\( a < d < c \)
\( b < c < d \)

\( a < d < a < b \)
Partially Ordered Sets:

Dilworth's Theorem

**Poset:** \((P, \leq)\)

- Set
- Binary relation

(i) Reflexive: \(a \leq a\).

(ii) Transitive: \(a \leq b \land b \leq c \implies a \leq c\).

(iii) Anti-symmetric: \(a \leq b \land b \leq a \implies a = b\).

(i) \(P = \{1, 2, \ldots, \delta\}\)

(ii) \(P = \{A_1, A_2, \ldots, A_m\}\)

\(\leq\) \(\subseteq\) \(a \leq b\) iff \(a \subseteq b\)

\(\leq\) \(\subseteq\)
\( a, b \) are comparable if \( a \leq b \) or \( b \leq a \)

Otherwise they are incompparable

\[ a < b \iff a \leq b \text{ and } a \neq b \]

Chain: \( a_1 < a_2 < \ldots < a_s \)

Anti-chain: \( a_1, a_2, \ldots, a_s \)

all pairwise incomparable.

\[
\left[ \ldots \right. \text{Sperner family} \left. \right] 
\]
$P$ is finite:

1. $m_1 = \text{max. size of a chain}$

2. $m_2 = \text{max size of anti-chain}$

3. $C_1, C_2, \ldots, C_m$ are chains, $C_1 u \ldots C_m = P$

   minimize

4. $A_1, A_2, \ldots, A_n$ are anti-chains, $A_1 v \ldots A_m = P$