1. Consider the following game: There is a pile of \( n \) chips. A move consists of removing any proper factor of \( n \) chips from the pile. (For the purposes of this question, a proper factor of \( n \), is any factor, including 1, that is strictly less than \( n \)). The player to leave a pile with one chip wins. Determine the \( N \) and \( P \) positions and a winning strategy from an \( N \) position.

2. Consider the following game: There is a single pile of \( n \) chips. A move consists of removing (i) any even number of chips provided it is not the whole pile, or (ii) the whole pile, but only if it has 2 (mod 3) chips. The terminal positions are zero and one. Determine the Sprague-Grundy numbers of each pile size. (Compute the first 15 numbers and see if you can see a pattern.)

3. Consider the following multi-pile game. A move consists of either (i) removing one, two or three chips from any pile, or (ii) splitting a pile of size \( n \geq 2 \) into two piles of sizes 1 and \( n - 1 \). Determine the Sprague-Grundy numbers for a game that starts with a single pile of size \( n \). (Compute the first 15 numbers and see if you can see a pattern.)

Suppose that the current position consists of three piles of sizes 3, 5 and 7. Show that this is an \( N \)-position and find all of the winning moves.