1. Let \( \alpha \in \mathbb{R} \) be given and put \( S = \{ x \in \mathbb{Q} : x < \alpha \} \) be given. Show that \( \sup(S) = \alpha \).

2. Use the definition of limit to show that \( \frac{5}{n^2} \to 0 \) as \( n \to \infty \).

3.* Use the definition of limit to show that \( \frac{2n}{n+1} \to 2 \) as \( n \to \infty \).

4. Let \( \{x_n\}_{n=1}^{\infty} \) be a real sequence and \( l \in \mathbb{R}\setminus\{0\} \) be given. Assume that \( x_n \neq 0 \) for all \( n \in \mathbb{N} \) and that \( x_n \to l \) as \( n \to \infty \). Show that \( \frac{1}{x_n} \to \frac{1}{l} \) as \( n \to \infty \).

5.* Let \( \{x_n\}_{n=1}^{\infty} \) and \( \{y_n\}_{n=1}^{\infty} \) be real sequences. Show that if \( \{x_n\}_{n=1}^{\infty} \) is bounded and \( y_n \to 0 \) as \( n \to \infty \) then \( x_ny_n \to 0 \) as \( n \to \infty \).

6.* Let \( \{x_n\}_{n=1}^{\infty} \) be a real sequence and \( l \in \mathbb{R} \) be given. Define the real sequence \( \{y_n\}_{n=1}^{\infty} \) by

\[
y_n = \frac{1}{n} \sum_{k=1}^{n} x_k \quad \text{for all } n \in \mathbb{N}.
\]

Show that if \( x_n \to l \) as \( n \to \infty \) then \( y_n \to l \) as \( n \to \infty \).

7. Determine whether or not \( \{\sqrt{n^2 + n} - n\}_{n=1}^{\infty} \) is convergent. If it is convergent find the limit.

8.* Let \( l \in \mathbb{R} \) be given. Show that there is a sequence \( \{r_n\}_{n=1}^{\infty} \) such that \( r_n \in \mathbb{Q} \) for every \( n \in \mathbb{N} \) and \( r_n \to l \) as \( n \to \infty \).

* Problems marked with an asterisk should be written up and handed in.