21-111 Calculus I - Fall 2004

First Course Test

September 27, 2004

Name:

Recitation Group:

There are 5 problems on this exam. Complete all problems, showing all work. Extra space is given on page 7. Please indicate clearly if you use the extra space. Good luck.

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1. [ 16 points ] Given

\[ f(x) = x + \frac{1}{x} \text{ for } x \geq 1 \]
\[ g(x) = 1 + x^2 \text{ for } -\infty < x < \infty \]

find the functions

\( (f(g(x)), (g \circ f), (f(f(x))) \)

if they exist. Make sure to specify the domains of the functions.

**Solution:**

\( f(g(x)) = 1 + x^2 + \frac{1}{1 + x^2} \) with domain \( x \) any real number (as \( 1 + x^2 \geq 1 \) for all \( x \)).
\( g(f(x)) = 3 + x^2 + \frac{1}{x^2} \) with domain \( x \geq 1 \).
\( f(f(x)) = \frac{(x^2 + 1)^2 + x^2}{x(x^2 + 1)} \) with domain \( x \geq 1 \).

[ 4 points ] Evaluate \( (f(g(t^2))) \) and \( [(f(g(2)))]^2 \).

**Solution:**

\( f(g(t^2)) = 1 + t^4 + \frac{1}{1 + t^4} = \frac{2 + 2t^4 + t^8}{1 + t^4} \)
\( [(f(g(2))]^2 = 27 \cdot \frac{1}{25} = 26 \frac{2}{25} \)

2. [ 15 points ] What is the value after 4 years of $600 invested at 6% annual interest compounded twice a year? [Hint: \((1 + 0.03)^8 \approx 1.27.]\)

**Solution:**

\( 600(1 + 0.06)^4 \cdot 2 = ($)762 \)

3. A factory produces \( N(t) = 3t - 3 \) robot arms after \( t \) hours of operation. The fixed cost of production is $5000 and the cost to produce one robot arm is $700. The revenue for selling \( x \) robot arms is \( R(x) = 800x - 200 \).

(a) [ 8 points ] How many arms must the company make and sell for the revenue to equal the cost?

**Solution:** \( R(x) = C(x) \) when \( x = 52 \).

(b) [ 10 points ] Find a function representing the profit made by operating the factory for \( t \) hours (Assume all the arms manufactured are also sold.) **Solution:** \( P(t) = 300t - 5500 \)

[ 2 points ] Find the initial profit or loss \( (t = 0) \). At what rate does the profit increase in each hour? **Solution:** Initial loss \( P(0) = -5500 \) and increase per hour $300.


4. A corporation builds printers. After operating for \( t \) months, their factory in Bangkok produces

\[ B(t) = 300t \]

printers. In the same amount of time, their factory in Madrid produces

\[ M(t) = 100t + 10t^2 \]

printers. The revenue generated from selling \( x \) printers is \( R(x) = 200x - 300 \) dollars.

(a) [10 points] After how many months of operation have the factories produced the same number of printers? **Solution:** \( B(t) = M(t) \) when \( t = 0 \) or \( t = 20 \).

(b) [10 points] What is the revenue generated for the corporation by these factories after \( t \) months of operation? **Solution:**

\[ R(B(t) + M(t)) = 80,000t + 2000t^2 - 300 \]

5. Simplify the following, leaving only one fraction and no negative exponents.

(a) [8 points] \[ \frac{xy}{x^3-y^3} \cdot \frac{x-y}{x} \] **Solution:** \[ \frac{1}{x-y} \]

(b) [10 points] \[ \frac{x}{y} - \frac{1}{x} \cdot \frac{y}{x^2} + \frac{x}{2y} \] **Solution:** \[ \frac{2(x^2-y)xyz-(x-y)(x^2+y^2)}{(x^2+y^2)xyz} \]

(c) [7 points] \[ \left( \frac{4x^{-8}}{9y^6} \right)^{-\frac{1}{2}} \] **Solution:** \[ \frac{3}{2}y^3x^4 \]