Supplementary Problems 1*, 2*, and 3*.

To clarify what is asked in problems 1–16 on p. 430: you need to determine both the radius and the interval of convergence (you may use $x$ instead of $z$ if you want). You should be able to do these problems efficiently. Write up numbers 3*, 5*, 6*, and 10*.

You should be able to problems like 1–10 on p. 438.

Supplementary problems

Problem 1. Suppose that the function series $\sum_{n=1}^{\infty} |f_n(x)|$ converges uniformly on $S$. Prove that the series $\sum_{n=1}^{\infty} f_n(x)$ converges (1) pointwise on $S$; (2) uniformly on $S$.

Problem 2. Suppose that the function series $\sum_{n=1}^{\infty} f_n(x)$ converges uniformly on $I = (a, b)$. Suppose that for some point $x_0 \in I$ we have $\lim_{x \to x_0} f_n(x)$ exists for all $n \in \mathbb{P}$ (but the functions are not necessarily continuous at $x_0$). Prove or disprove that

$$\lim_{x \to x_0} \sum_{n=1}^{\infty} f_n(x) = \sum_{n=1}^{\infty} \lim_{x \to x_0} f_n(x).$$

Problem 3. Find the limit

$$\lim_{x \to +\infty} \sum_{n=1}^{\infty} \frac{x^2}{1 + n^2 x^2}.$$

Be sure to carefully justify your manipulations.