Fall 2019 An introduction to convex and discrete geometry 21-366 CMU

Lecture: MWF 11:30 – 12:20 pm, Porter Hall A18A

Lecturer: Tomasz Tkocz, Wean Hall 7206, `ttkocz@math.cmu.edu`

Office Hours: ...... or by email appointment

Course website: `http://math.cmu.edu/~ttkocz/teaching_1920.php`

Course description: Convex geometry is about convex sets in Euclidean spaces, which are among simplest geometric objects and yet posses a rich structure governed by interesting and often surprising phenomena (especially in high dimensions). Discrete geometry is mostly concerned with finite sets of points, lines, planes, convex sets etc. and the focus is on their combinatorial properties. This course offers an elementary introduction to topics such as basics of convexity, duality, polytopes, elements of the Brunn-Minkowski theory, etc. Another goal is to develop a variety of techniques, including algebraic, analytic, combinatorial and probabilistic methods useful in solving concrete classical and modern geometric problems such as isoperimetry, coverings, number of incidences, or almost isometric embeddings.

Prerequisites: Basic background in linear algebra, analysis and probability.

Literature:

- Ball, K., An elementary introduction to modern convex geometry. Math. Sci. Res. Inst. Publ., 31, *Cambridge Univ. Press, Cambridge*, 1997.
- Chakerian, G.D., Sangwine-Yager, J.R., Theory of Convex Sets.
- Leonard, I. E., Lewis, J. E., Geometry of convex sets. *John Wiley & Sons, Inc., Hoboken, NJ*, 2016.
- Matoušek, J., Lectures on discrete geometry. Graduate Texts in Mathematics, 212. *Springer-Verlag, New York*, 2002.
- Matoušek, J., Thirty-three miniatures. Mathematical and algorithmic applications of linear algebra. *American Mathematical Society, Providence, RI*, 2010.

Course content: basic convexity, Carathéodory's theorem, separation, extreme points, combinatorial convexity (Radon's theorem, Helly's theorem, centrepoint), duality, basics of polytopes, the moment curve, arrangements and incidences (the Szemerédi-Trotter theorem), coverings by strips (the plank problem), volume (Brunn-Minkowski's inequality), elements of asymptotic convex geometry and high dimensional phenomena, examples of algebraic methods (equiangular lines, equilateral sets, Borsuk's question)

Learning objectives:

- understanding the structure of convex sets in Euclidean space
- analytic, algebraic and probabilistic techniques with applications in geometry
- applications of convexity based arguments to other areas

Course oragnisation: There are three lectures per week. We follow rather closely the textbooks mentioned above. Lecture notes will be regularly uploaded on the course website. There are fortnightly assignments. There is one midterm and the final take-home exam.

Homework: This is the essential part of the learning process in this course. Simply listening in class or reading texts is not sufficient. Understanding mathematics requires practice. The course will be fast-paced, therefore homework assignments will help you study systematically, without gaps in comprehending the material.

Fortnightly assignments will be posted on the course website. The assignments will be collected in class, *before* the lecture begins. Late submissions will not be accepted. The lowest homework score will be dropped. You may lose points for poor presentation. Please write neatly and provide complete solutions, all explanations and arguments, not just answers. Plagiarism is not tolerated.

Exams: There will be one 50 min midterm exam taken during class: **16th Oct**. There will be a take-home final exam covering all the material. Exam questions will mainly be homework-style problems.

Books, notes, or any electronic devices (including calculators) will not be allowed in exams.

Grades: The midterm grade will be based solely on the midterm exam. The final grade will be based on homework, midterm and the final exam, computed as a weighted average: 40% Homework + 30% Midterm + 30% Final exam