

Please write down your name in CAPITAL letters. Please write down your solutions to each problem on an individual sheet. No resources allowed (books, notes, electronic devices, etc.)

1. How many nondecreasing functions $f: \{1, 2, \dots, 2019\} \rightarrow \{1, 2, 3, 4, 5\}$ are there? Let f be such a function selected uniformly at random. What is the probability that $f(1) = 2$? What is the probability that $f(1010) = 3$?
2. There are 20 interesting restaurants in Pittsburgh. For 7 consecutive days, every night we dine at a randomly selected interesting restaurant. What is the expected number of restaurants we have tried?
3. Let X and Y be independent identically distributed discrete or continuous random variables such that $\mathbb{E}X^2 < \infty$. Show that $\text{Var}(X) = \frac{1}{2}\mathbb{E}(X - Y)^2$.
4. Let X and Y be independent identically distributed random variables taking at most two different values. Show that $\mathbb{E}|X + Y| \geq \mathbb{E}|X - Y|$.