Fall 2004 Practice Exam Version II

Allow yourself 55 minutes for this practice exam. You may use Table 6.2.1 in the text (page 319), which will be provided for Exam 3. The following information will also be provided:

\[
\int x \cos(ax) \, dx = \frac{1}{a^2} \cos(ax) + \frac{x}{a} \sin(ax) + C
\]

\[
\int x \sin(ax) \, dx = \frac{1}{a^2} \sin(ax) - \frac{x}{a} \cos(ax) + C
\]

In case the $2 \times 2$ matrix $A$ has complex eigenvalues $r = \alpha + i\beta$ and $\tau = \alpha - i\beta$, and $\xi = u + iv$ is an eigenvector corresponding to eigenvalue $r$, the general solution to the system $x' = Ax$ is

\[
x(t) = e^{\alpha t} \left[ c_1 (u \cos \beta t - v \sin \beta t) + c_2 (v \cos \beta t + u \sin \beta t) \right]
\]

No calculator of any kind is permitted. Show all work and give clear explanations. Remember that the purpose of an exam is for you to showcase your mastery on conceptual as well as computational levels.

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1. (25 Points) Solve the following initial value problem using Laplace transform methods.

\[ y'' + 4y' + 13y = 0, \ y(0) = 1, \ y'(0) = -5 \]
2. (25 Points) Find the function \( f = f(t) \) whose Laplace transform is

\[
F(s) = \frac{1 - e^{-2s} - 2se^{-3s}}{s^2}
\]

and sketch the graph of \( f \), for \( t \geq 0 \).
3. (25 Points) Determine the general solution to the system

\[ x' = \begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix} x \]

Provide a sketch which makes it clear how all trajectories behave as \( t \to \infty \).
4. (25 Points) Let $f$ be identically equal to 1 on the interval $[0, 3]$. Find a Fourier sine series for $f$, valid at least on $(0, 3)$. 