21-110 Problem Solving and Recreational Mathematics
Project Guidelines and Suggested Projects
Spring 2008

The purpose of each problem-solving project is twofold: (i) To give *you* the chance to be a mathematician by using whatever knowledge you happen to have, and whatever strategies may occur to you, to tackle a challenging problem; and (ii) Once you've solved the problem, to give a clear, thorough, step-by-step *explanation of the solution strategy.* You should write up your results as though *you are the expert* on the problem, and give a clear exposition which will be understandable to *non-experts.*

When you write up your results, you'll want to hold your reader's hand throughout the entire solution process. You might try and give the discussion the feel of an article in a mathematics magazine, or something like that. One suggestion I could offer is to hand a draft to your roommate or whomever and ask them to read through it and see if they follow it (once they've read the problem statement, of course). If they report that they got stuck at certain points and were momentarily confused, then you know that those are the best spots to offer a bit more explanation.

Another suggestion is to make believe that the problem you've chosen is famous and notoriously hard and has gone unsolved for years and years. Imagine that you are the first person to solve it. So the world will be very anxious to learn how you did what they all tried in vain to do. So you will have to guide them through the process.

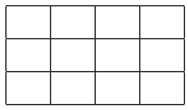
The items at the end of the list are designated with (*) and are writing assignments rather than problems. You may do only *one* such project; the other two must be problem-solving projects. (Or you may do three problem-solving projects.)

In addition to the items on this list, you may have an idea which may be suitable for a problem-solving or writing project. If so, *you must check with me first* to make sure that what you have in mind will fulfill a project requirement.

Each project counts for 20% of the final grade. Final drafts of all projects must be turned in by class time on Friday, May 2, *and may be turned in on any day before that.* One rough draft of each project must be submitted for me to review and critique. I will return the draft to you as soon as possible with my comments and suggestions for revision.

Due dates for drafts: February 8, March 21, April 21. Final versions of all projects due May 2. (Drafts and final versions may be submitted early.)

1. Given an $m \times n$ "chessboard", formed by m equally spaced horizontal line segments and n equally spaced vertical line segments (a 4×5 chessboard is shown below), how many squares are determined by the line segments? (The 4×5 chessboard has 20 squares: 12 with an area of 1 square unit, 6 with area 4 square units, and 2 with area 9 square units.)



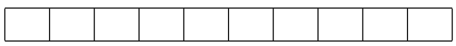
The following problem is taken from *A Mathematics Sampler: Topics for the Liberal Arts* by W.P. Berlinghoff, K.E. Grant, and D. Skrein.

2. In a hidden monastery, embedded in the floor of a room are three silver spikes. On one of them there are 50 gold disks, shaped like large washers, which are graduated in diameter so as to form a conical tower. The monks work tirelessly at the task of transferring the disks to re-form the tower on a different spike, according to the following rules:

   - Only one disk at a time may be moved.
   - Each disk must be on one of the spikes except when being moved.
   - A larger disk may not be placed on top of a smaller one.

   What is the minimum number of moves that the monks must make to complete their task? Assuming that they make no wasted moves and move one disk every second, how long will it take them to finish?

3. Suppose a ten-digit number is to fill the boxes below:



0 1 2 3 4 5 6 7 8 9

   Let's call the box with a zero below it "Box 0"; call the box above the numeral 1 "Box 1", and the next one "Box 2", and so on. Find a ten-digit number whose digits full the boxes in the following way: The digit in Box 0 gives the number of times 0 occurs as a digit in the number; the digit in Box 1 gives the number of times 1 occurs as a digit in the number; the digit in Box 2 gives the number of times 2 occurs as a digit in the number, etc. So, for example, if there were a 7 in Box 5, this would mean that 5 would occur seven times, i.e., seven of the ten digits would be 5. (An example of such a number, then, might be $5,855,755,055$. But actually, this particular number is unsatisfactory because, for example, there is a 0 in Box 7, which should mean that 7 does not occur as a digit.)

As with all of these projects, what is of interest is your *problem solving strategy*, so you cannot get credit for simply turning in a sheet of paper with a ten-digit number on it. You must explain the process whereby you discovered an appropriate number.

Items 4 through 7 are taken from *Mathematics: Problem Solving through Recreational Mathematics*, by Bonnie Averbach and Orin Chein.

4. In order to select capable contestants for their quiz shows, some producers require prospective contestants to compete off camera. One show, "Fractured Funnybones," usually chooses four candidates and conducts a series of head to head matches between each pair of them. The candidate who performs the best is then chosen to appear on the show.

   The game is played as follows: The contestants are shown a key word, and whoever first thinks of a joke using that word in the punch line gets one point. The first player to get five points wins.

   When Manny Morris, Nancy Novokov, Patti Proctor, and Thomas Twickenham competed against each other, the results were as follows:

   Patti won all three of her games and her opponents scored a total of only 2 points against her.

   Nancy scored a total of 10 points and her opponents also scored 10.

   Manny and Thomas both scored 8 points, but Manny allowed his opponents 15 whereas Tom's opponents scored only 14. In addition, Tom scored more points against Patti than Manny did.

   What was the outcome and the score of each head to head match?

5. Good poker players have four characteristics in common: They are familiar with the odds associated with card distribution; they know when it is wise to bluff; they have poker faces; and they are lucky.

   Angel invited four friends to play poker one evening, around a big circular table. They were Babs, Cleo, Dot, and Edie. The following was also true:

   Everyone was sitting next to someone who knew the odds, but four of the five were sitting next to someone who was not well versed in the probabilistic aspects of the game.

   Four of the people were sitting next to wise bluffers, but three of them were sitting next to someone who did not know when to bluff.

   Four of the players were sitting next to people with good poker faces, but everyone was sitting next to someone who could not keep a straight face.

   Exactly three of the people were sitting next to someone who was noted for good luck.

Each of the players had at least one of the desirable traits; but only one, the big winner of the night, had all four.

Babs knows the odds and knows when to bluff, but does not have a poker face and is not noted for good luck.

Edie is not sitting next to anyone who knows when to bluff.

The person on Dot's right has a poker face.

The person on Cleo's left does not know when to bluff.

Angel knows the odds, but is not lucky.

Who was the big winner of the evening, what was the seating arrangement, and which traits did each of the five players have?

6. Three women (Abby, Janice, Linda) and two men (Martin, Roberto) are a singer, a dancer, a comic, a television writer, and a theatrical agent, although not necessarily in that order.

   Abby said: I'm not the comic. The writer and the dancer are happily married. The singer and the agent are engaged to be married.

   Janice said: The singer is my cousin. The writer and the dancer are siblings. The comic and the agent share an apartment.

   Linda said: I am not the writer. The singer and the agent hate each other. The dancer and the comic frequently work together.

   Martin said: The singer owes me $10. The writer and the dancer are not related and have never met. The comic and the agent are next-door neighbors in an apartment house.

   Roberto said: The singer saved my life once. The agent lives alone in an old mansion. The dancer and the comic have never met.

   If the only certainty is that every statement in which an individual alludes to his or her own profession is true, who is who?

7. The magician placed an apple, a banana, a peach, and 25 matchsticks on the table. He then selected three volunteers from the audience. Their names were Alvin, Julia, and Melonie. The magician then handed one of the matchsticks to Alvin and two matchsticks to Julia, but he did not give any to Melonie. He was then blindfolded, and each of the volunteers was asked to remove one of the fruits from the table.

   "Whichever of you removed the apple should now remove a number of matchsticks equal to the number I handed you a moment ago. Whoever removed the banana should remove three times as many matchsticks as you were originally given. Finally, the remaining person should remove nine times as many sticks as I gave to him or her."

   "Now, tell me how many sticks remain on the table."

"Seven," was the reply.

"Then Alvin took the peach, Julia took the banana, and Melonie took the apple," declared the magician.

Naturally, he was correct.

Explain how this trick works.

The following problem is due to A. Gottlieb, "Puzzle Corner", *Technology Review*, January 1971.

8. There are five houses in a row (east to west), each of a different color and inhabited by men of different nationalities, with different pets and preferences in beverages and cigarettes.

   The Englishman lives in the red house.

   The Spaniard owns the dog.

   Coffee is drunk in the green house.

   The Ukranian drinks tea.

   The green house is east of the ivory house and next to it.

   The Old Gold smoker owns snails.

   Kools are smoked in the yellow house.

   Milk is drunk in the middle house.

   The Norwegian lives in the most westerly house.

   The man who smokes Chesterfields lives in the house next to the man with the fox.

   Kools are smoked in the house next to the house where the horse is kept.

   The Lucky Strike smoker drinks orange juice.

   The Japanese smokes Parliaments.

   The Norwegian lives next to the blue house.

   Who drinks water? And who owns the zebra?

I have adapted the following problem from one which appears in *The Master Book of Mathematical Recreations*, by Fred Schuh.

9. The fair Freda is a temperamental princess whose love cannot be expected to last. One day Freda stood in the courtyard with a large number of suitors arranged in a circle around her. A guard at the entrance to the palace grounds remarked, somewhat anachronistically, "Musta been a hundred fellers came through that gate."

   Bartholomew stood to the left of Beauregard; each possessed charm and confidence in abundance, and each was sure he would win the heart of the fair Freda.

   The maiden began to move round the circle, starting with Beauregard and proceeding to his right, pointing at each suitor in turn and saying, "Love you. Love you not. Love you. Love you not ...." Each suitor to whom she said, "Love you not" dropped out of the circle and left the courtyard, dejected. Freda continued round and round the circle, until only Bartholomew was left standing ... whereupon she announced that she would marry Beauregard.

   The question is this: Can we improve upon the palace guard's estimate of the number of suitors in the original circle?

I have adapted the following problem from one appearing in *Principles of Mathematical Problem Solving*, by Martin J. Erickson and Joe Flowers.

10. Suppose Kukla, Fran, and Ollie are all very good at deduction and mental arithmetic, and each has a positive integer written on his or her forehead, so that they can see each other's numbers but not their own. Their teacher writes the numbers 11, 17, and 65 on the blackboard, and the three individuals are told that one of the numbers on the board is the sum of the three numbers on their foreheads. Kukla is asked, "Do you know your number?" If the answer is no, Fran is asked the same question, and then Ollie is asked. If all three answer in the negative, Kukla is asked again, then Fran, then Ollie, and so on in a cyclic pattern. Show that eventually, someone will answer, "Yes!"

    Once you have revealed how the game works with the numbers 11, 17, and 65, consider what will happen if the numbers are changed to some other combination of three numbers. Will it always happen that some player is able to determine the number on his/her own forehead? Or does this depend somehow on the triplet of numbers initially chosen?

The following problem is due to Kevin Purbhoo, *American Mathematical Monthly*, January 1998.

11. On a remote Norwegian mountain top, there is a huge checkerboard, 1000 squares wide and 1000 squares long, surrounded by steep cliffs to the north, south, east, and west. Each square is marked with an arrow pointing in one of the eight compass directions, so (with the possible exception of some squares on the edges) each square has an arrow pointing to one of its eight nearest neighbors. The arrows on squares sharing an edge differ by at most 45 degrees. A lemming is placed randomly on one of the squares, and it jumps from square to square following the arrows. Prove that the poor creature will eventually plunge from a cliff to its death.

I have adapted the following problem from one appearing in *Concrete Mathematics: A Foundation for Computer Science* by R.L. Graham, D.E. Knuth, and O. Patashnik. (You needn't be interested in computer science, though, to solve the problem.)

12. Given a real number s, define $[s]$ to be the *integer rounddown* of s; that is, the greatest integer which is less than or equal to s. (So for example, $[3.267] = 3$, while $[-4.1117] = -5$. For this problem, though, we will only be interested in positive values of s.) Suppose the Shootyerfoot Casino has a roulette wheel with 1000 slots, numbered 1 through 1000, and a game in which you bet \$1, and they spin the wheel. If the number N which comes up on the spin is divisible by $[\sqrt[3]{N}]$, then all players win and are given \$5. Otherwise, the house collects all the \$1 bets.

    Now of course, most casino games are designed so that the house has the advantage. Is that the case here? If so, explain why. If not, explain how the casino can modify the game (and you'll probably want to find the simplest modification(s) you can) so that the house does have the advantage.

13. Apply the principles of probability theory to the game of Yahtzee, by considering several scenarios which one might encounter during game play. (A fun way to start this project is to play several games of Yahtzee with your roommate and jot down such scenarios as you encounter them.) Examples might include the following:

    - Suppose, in some round, your first roll is 1, 2, 2, 3, 3. Is it "smart" to keep the twos and threes and try for a Full House (assuming you need it)? What is the probability that with two attempts at rolling one die you will roll a 2 or a 3, so that you get a Full House?
    - Suppose again that in some round you roll 1, 2, 2, 3, 3. If you keep 1, 2, 3 and roll the other two dice to try for a Small Straight or a Large Straight, what is the probability that you will succeed?

- Suppose you have only the Yahtzee and Three-of-a-Kind boxes left to fill, and you roll 5, 5, 6, 6, 6. Now you have a guaranteed Three-of-a-Kind worth 28 points. But by keeping the three sixes and rolling the other two dice (twice if necessary), you may get a Yahtzee. If you choose that strategy, what is the expected point value of the round?

You need not choose to investigate any of the scenarios above; perhaps you are more interested in others. Three scenarios is probably not enough; about five should do it. (If you wish, you might also include the results of a game in which you tested your findings, and comment on how well − or poorly − probability theory came through for you.)

14. Let us start with what we shall call the Five City Problem: We wish to start in Pittsburgh, travel to four other cities, and return to Pittsburgh, in such a way as to minimize the total distance travelled. The four other cities are St. Louis, New York, Atlanta, and Dallas. Mileages are given below:

	Atl	Dal	NYC	Pitt	St. L
Atlanta	*	795	841	687	541
Dallas	*	*	1552	1204	630
New York	*	*	*	368	948
Pittsburgh	*	*	*	*	588

Intuitively, one expects that due to the relative geographic locations of the cities, the shortest circuit will be as follows: Pittsburgh to New York, New York to Atlanta, Atlanta to Dallas, Dallas to St. Louis, and St. Louis to Pittsburgh. (Or one may travel the same circuit in reverse order.) You should verify, however, that there are 12 possible circuits, and that the one just given really is the shortest.

Now comes the fun part: Suppose a sixth city is added to the list, and consider the Six City Problem. You may choose the new city to be whatever you like, but to make things interesting, you should not choose a city which is directly between two of the above cities. Verify that there are now 60 possible circuits, but for heaven's sake, do not do this by listing all of them! Much too tedious. Try to find a slicker way to "count" the number of circuits.

Now since there are 60 possibilities, it would be nice to have a method available which would find the shortest circuit without using a "brute force" method, i.e., listing all 60 of them and comparing their lengths.

Before you look for such a method, though, see if you can determine how many circuits there would be in a Seven City Problem, an Eight City Problem, etc., and try to determine a formula for the number of circuits in an n-City Problem, where n is an unknown positive integer. You should find that the number of circuits grows by staggering amounts as the number of cities increases.

Next imagine that you have a supercomputer which will check circuits for you, at the rate of, say, a billion circuits per second. How many cities would lead to so many circuits that the computer could not succeed in checking them all during your lifetime? Once you've answered this question, it should be obvious that for even fairly modest City Problems, checking all possible circuits is simply not an option.

So your next move should be to try and determine some *algorithm*, i.e., a step-by-step general procedure, which, given any number of cities and the mileages between them, finds a . . . well, *pretty good* circuit. It may not be guaranteed to be the *shortest* circuit possible, but, by some criteria which you will need to explain, the circuit identified by your algorithm is . . . mmm, not bad. You should give some examples of your method applied to Five City Problems (different from the original one above), with fabricated data if you like, and show how your method *sometimes* gives the best circuit, but sometimes does not. The algorithm should be *blind to geography*, which is to say that your method should *not* take into consideration the relative positions of the cities; it should only use the mileage data.

Finally, once you have decided on a strategy for finding not-too-shabby circuits, put it to a *real* challenge: Take the five cities above and add 10 more, of your choice, for a Fifteen City Problem. (To make things interesting, try to choose the cities so that they are not arranged in some "nice" pattern, i.e., circular.) Again, the goal is to start in Pittsburgh, hit each of the other 14 cities once, and return to Pittsburgh, minimizing the total distance travelled. In your write-up, include a complete mileage table for the 15 cities involved.

Determine the circuit chosen by your algorithm, and indicate the total mileage. Now consult a map and see if your solution seems intuitively good from a geographical standpoint. Using your map, trace ten other circuits which seem to you as though they would be strong candidates for the shortest. Then refer to the mileage table to compute the lengths of these other ten circuits. Tabulate these lengths and compare them to the circuit which your algorithm chose. How do they compare? How well did your algorithm do? If it didn't do so well, include a discussion of what may have gone wrong.

Incidentally, if you can develop an algorithm which is *guaranteed* to find the shortest circuit (without checking all of them), given any number of cities, it should be good for a Ph.D. thesis in mathematics or computer science, as well as worldwide fame; no such algorithm is known.

15. From “The Jogger’s Problem”, R.S. Bird, *Information Processing Letters*, **13**, No. 3, December 1981, pages 114-117: “Consider the plight of a particularly reluctant jogger who, while willing to undertake exercise, wishes to make it as least unpleasant as possible. The jogger is confronted with a network of roads each of which possesses some positive measure of undesirability. Beginning at a designated point the jogger wishes to plan a circular route, no road being traversed more than once, of minimum total undesirability.” Give some thought to this problem, in the case that the “reluctant jogger” wishes to start at home and traverse some minimum total distance, such as five miles. Observe that one of the constraints of the problem is that the jogger may not run down any given stretch of road more than once, and each stretch of road (say from one intersection to another) is rated according to its unpleasantness, by such factors as, let’s say, muddiness, steepness, traffic, unrestrained dogs, or what have you.

    As you search for an efficient strategy to solve the problem, I would recommend that you draw for yourself quite a lot of “maps” which have different characteristics; the goal is to look for a strategy that would apply to all of them and, if possible, to any “map” that one might consider. (I put the term “map” in quotes because you may be able to work with some more simplified diagram; if so, you should show how to convert an actual map into your simpler diagram.)

    To simplify your investigation, you may want first to consider only the types of “undesirability factors” which would not depend on the *direction* of travel; for instance, a muddy road is muddy regardless of which way the jogger is running, but a steep road is steep only in one direction. Once you have decided on an efficient strategy for handling the jogger’s problem with direction-independent undesirability factors, see if you can modify your strategy to include steepness as a factor. (Remember, however, that if steepness is included, the jogger must return to the original elevation by the time he/she gets back home.)

    Something else to think about: If we stipulate that the jogger wishes to run at least five miles, there is the possibility that the most unpleasant route is also the *shortest* (let’s say exactly five miles), while the least unpleasant route is the longest (thirty miles or some outrageous length). So you may want to cook up an example where this sort of outcome results, and perhaps present a modified strategy that attempts to guard against circuits which are too long.

    This is a very open-ended problem, so if you choose it, I would suggest you meet with me to discuss it a bit after you’ve worked on it some. It’s probably also a good idea to show me your examples before you write up your first draft.

16. In class, we have studied (or are studying, or will study) the following five election methods:

    - The Plurality Method
    - The Borda Method
    - The Condorcet Method
    - The Plurality-with-Elimination Method
    - The Plurality-with-a-Runoff Method

    We have also considered four fairness criteria for election methods. They are

    - The Majority Criterion
    - The Independence-of-Irrelevant-Alternatives Criterion
    - The Condorcet Criterion
    - The Monotonicity Criterion

    Write a paper in which you study any three of the five election methods above. For each of the three methods you choose, do the following:

    (a) Explain how the method works in practice.

    (b) Give examples of the method applied to particular preference schedules. You must create your own examples.

    (c) Explain whether the method satisfies or violates each of the fairness criteria. (No method will satisfy all four of them.) In each case, if the method satisfies the criterion, you must establish that fact with a general argument; if the method violates the criterion, you must provide a specific example to show how the violation may occur.

    The paper should be self-contained and not assume that the reader already knows about election methods or fairness criteria. This will mean, for example, that you will need to provide explicit statements of the fairness criteria as part of your write-up.

Problems 17 and 18 were adapted from the 2002 Emmy Noether High School Mathematics Day at Texas Tech University.

17. At 3:00 the hands of an analog clock form an angle of 90 degrees. At 9:00 a 90-degree angle is formed once again. What angle do the hands form at 7:38? Choose two other clock times and determine the angle formed by the hands at those times. (Your two examples should be interesting and nontrivial, not straightforward and simple.) Then discuss (i) all possible clock times for which the hands form a 90-degree angle, (ii) all clock times

for which the hands are lined up perfectly, thereby forming a 0-degree angle, and (iii) all clock times for which the hands point in diametrically opposite directions, thereby forming a 180-degree angle.

18. The natural numbers (i.e., the "counting numbers" 1, 2, 3, 4, etc.) are printed on a ribbon with one number immediately following the other, like so:

$$123456789101112131415\ldots.$$

The ribbon stretches due north from Pittsburgh, all the way to the north pole, then southward on the other side of the globe, all the way to the south pole, and then back north to Pittsburgh, where the ribbon is attached to itself. Each digit requires 2 centimeters to print. What is the last complete number printed on the ribbon?

Problem 19 is inspired by the article "Lost in a Forest", by Steven R. Finch and John E. Wetzel, in *The American Mathematical Monthly*, **111**, No. 8, October 2004.

19. Imagine that you are stranded in the ocean, in a dense fog at night, some unknown distance from a straight shoreline. Assume that you can stay afloat, and swim, forever (without sleeping or eating). The challenge is to swim to shore. Assume that you have a compass and some device that will measure distances (electronically, let's say).

Obviously you would like to swim directly toward the shore, in a direction perpendicular to the shoreline. But without being able to see the shore (even if you are very close), you will have no idea which direction that is. So of course it is ill-advised to just pick some direction and swim off in a straight path, because you may never reach the shore. So you will need some strategy which entails changing direction (at least once).

One strategy might be to swim westward, say, some distance d units. (The units could be feet, or miles, or whatever; it doesn't matter, as you'll see.) Then you could turn northward, and swim the distance d. Then you can turn eastward, swimming a distance $2d$ units, then go south $2d$ units, then west $3d$ units. Then north, east, south, west, etc., so that your path is a "square spiral". This way, you keep widening out from your starting point, and eventually you will have to hit the shoreline.

But is this the "best" strategy? To answer that question, you would have to formulate some criterion (or criteria) by which one strategy is "better than" another. For instance, other types of "spirals" would work; instead of making each turn a 90 degree angle, you could turn at a 135 degree angle each time, starting out going west, then turning northwest, then north, then northeast, etc.

The latter strategy is fortuitously more successful if the shoreline happens to lie north of your starting position, close enough so that your first northward leg takes you right to shore. But the latter strategy is worse if the shoreline is very close to you, but to the south. So obviously, if you compare any two strategies A and B, you may find that A is "better than" B in particular cases, and B is better than A in others, depending on locations and distances. But are there criteria by which you can assert that one strategy is *generally better* than another? Is there a way to say, "This strategy is *more likely* to get you to shore with less swimming than that strategy?"

So this project is to investigate such questions. The strategies you could conceive are of course limitless. So experiment with some, and then discuss some. Discover for yourself ways in which you can reasonably compare them. (To open up more possible strategies, you may assume that it is possible to swim in a "curved spiral", and/or swim along a path that traces out a circle, or part of a circle.)

In addition to the scenario above, wherein the distance to the shoreline is not known, also consider the scenario in which is the distance to shore *is* known. (You just don't know the *direction* to shore.) Is this new information useful in any way? Does this have any effect on the recommended strategy or strategies?

Have fun! There are no "right" or "wrong" answers on this one; only "good" or "not altogether satisfying" answers.

20. Write a paper on the mathematics behind *cryptography* (or cryptology), the science of making and breaking codes. If you enjoy prime numbers, or number theory in general, then you should find cryptography interesting. Your paper should discuss four or five well known cryptographic schemes (that is, types of codes) and give original examples (*this means examples that you design yourself and explain*) of some simple ciphers (coded messages) of each type. You should also indicate some of the common applications of effective encoding; a brief history of the subject might also be in order.

    Optional: If you have an interest in true crime, you may also want to include an account of a notorious and quite unexpected instance of encryption. In the late 1960s and early 1970s, a serial killer calling himself "The Zodiac" was operating in the San Francisco area. Along with a number of taunting letters to the Bay Area newspapers, he sent a 408-character coded message, claiming, "In this cipher is my identity." If you know anything about criminal profiling, then you will realize the urgency with which such a message would need to be decoded, not only to determine the actual content of the message, but also to guage the level of sophistication of the code, thereby providing some clue as to what type of person would be capable of designing it. A comprehensive treatment of the Zodiac case is given in Robert Graysmith's book *Zodiac*.

The code was cracked, but the Zodiac murders are still unsolved.

21. (*) Do some research on early attempts to make the notion of time quantitative. When did the human race become interested in developing units for measuring time, and why? To what extent were early attempts to measure time successful? (Or is the last question even fair? Do we have much different criteria for *successful measurement* of time than our ancestors did?) Write a paper addressing these questions, and use the answers to advance your opinion either that *time* is an actual natural/physical entity, or that it is a mathematical artifice. (In attempting to find sources for this project, you may have little success searching under *time*; the keyword *calendars* may be more helpful. Philosophers and sociologists have been known to ponder this question, too, so you may be able to get some help there.)

22. (*) Read *Flatland*, a classic short novel written in 1884 by a theologian named Edwin A. Abbott. The narrator of the story is a square who lives in a two-dimensional world whose inhabitants' social stations in life vary according to their geometric shapes. The square is visited by a sphere from a 3-dimensional world, who appears to him as a circle since the square can only see that part of the sphere which is intersecting the plane of "Flatland". By and by the sphere is able to convince the square of the existence of a third dimension which the square has never experienced.

    Review this story, commenting first on its effectiveness in communicating mathematical ideas (you might offer any insights you gained from reading). But also discuss the value of the story as a work of theology and/or social commentary, providing examples from the narrative of how Dr. Abbott used his cute little story of scientient geometric shapes to satirize the prevailing opinions of his day.

23. (*) Write a paper discussing the contribution of mathematics, and of mathematicians, to society. What is it, exactly, that mathematicians do for the species as a whole? What is the role of the mathematician in society? Why should we value this particular specialty? Often people are surprised to find that some academics do research in mathematics. They ask, "How can one do mathematical *research*?" They seem unable to conceive that there can be *new discoveries* and *new knowledge* generated in mathematics. Also, we often see news items concerning new advances in medicine, archaeology, even theoretical physics; but new advances in mathematics attract very little attention. Nor do we usually see documentaries which spotlight the career of a prominent mathematician. So why is that?

24. (*) This project is similar to the one above and is for movie buffs: Write a paper on mathematics and mathematicians in film. You must watch at least three movies and critique their portrayal of the mathematician character. The movie *A Beautiful Mind* is based (loosely) on the book by the same name (see Project 26) and on the life of John Nash. So the mathematician in this movie is not fictional (although many of the events depicted are). All of these movies include fictional mathematicians: *Pi, Good Will Hunting, Sneakers, The Mirror Has Two Faces.*

    Discuss whether you think these movies have any impact on the general public's conception of (1) mathematicians as people, and/or (2) what role mathematicians play in society.

25. (*) Write a paper on the history of one of the most elusive theorems of mathematics, Fermat's Last Theorem, named for Pierre de Fermat (1601-1665), a French lawyer who pursued mathematics as a hobby. Fermat studied the equation $x^n + y^n = z^n$ and asserted that no positive integers x, y, and z satisfy this equation when $n > 2$. To be more exact, in 1637 he wrote the following in the margin of a book: " . . . it is impossible to separate a cube into two cubes, a fourth power into two fourth powers, or, generally, any power above the second into two powers of the same degree: I have discovered a truly marvelous proof which this margin is too narrow to contain." Unfortunately, Fermat never left behind a copy of his "truly marvelous proof," and for over three centuries, mathematicians endeavored to find proofs of their own, a number of which were announced and even published before being found to be flawed. Finally, in 1993 Princeton University mathematician Andrew Wiles announced that he had proved the result. You may be able to find news items concerning Wiles' announcement even in popular periodicals (*The Wall Street Journal* ran a story). It turned out, though, that Wiles' original proof was inadequate, so he and his former student, Richard Taylor, took some time to repair it. For this chapter in the story, see *Notices of the American Mathematical Society* (which you can find in the E & S library), volume 40 (1993), pages 575-576; volume 41 (1994), pages 185-186; volume 42 (1995), page 48; and volume 44 (1997), pages 346-347.

    For your research, you will want to consult some biographies of Fermat which were published before 1993 in order to get a sense of the tantalizing mystery shrouding his Last Theorem; hopefully you can also find accounts of unsuccessful attempts to prove the result. (You needn't restrict your search to sources which exclusively concern Fermat; virtually any comprehensive history of mathematics will contain a chapter on Fermat or his era.) Finally, you should summarize the story of how Wiles and Taylor put the problem to rest. A possible source here is the book *Fermat's Enigma* by Simon Singh.

26. (*) Read Paul Hoffman's book *The Man Who Loved Only Numbers* and write a biographical paper on Paul Erdos (1913-1996), a fascinating character whose mathematical research so consumed him that he couldn't be bothered to own a home, have a family, or even take a job! "Property is nuisance", he said. His life was an optimization problem: How can one maximize the amount of time spent doing mathematics, while minimizing the time spent ... well ... doing *anything else*?

    Erdos was one of a kind, a scholarly nomad who was welcomed everywhere he went (which is all over the world) because he was so intellectually stimulating and because he was a genuinely nice fellow. He would arrive at each new destination declaring, "My brain is open!"

    Your paper should outline Erdos' life and also include some discussion of his scholarly work and its significance.

27. (*) Read Sylvia Nasar's excellent book *A Beautiful Mind* and prepare a condensed biography of John F. Nash, a remarkable mathematician born in West Virginia in 1928, who battled schizophrenia for some 30 years. The blurb on the jacket of Nasar's book reads, "A legend by the age of thirty, recognized as a mathematical genius even as he slipped into madness, John Nash emerged after decades of ghostlike existence to win a Nobel and world acclaim." In 1994, Nash won a Nobel prize in economic science for a paper he wrote in 1949, before he had completed his doctoral thesis!

    Your paper should outline the unusual trajectory of Nash's life and also give some treatment of his scholarly work and its significance. You may also wish to watch the 2001 movie, *A Beautiful Mind*, and comment on how well (or poorly) the film conveys Nash's life story. (Nash obtained his undergraduate degree at CMU. He is still alive and lives in Princeton Junction, New Jersey.)