21-737 Probabilistic Combinatorics
Fall 2025 MWF 2:00-2:50 Wean 8220
Tom Bohman

- **Course description.** This course covers the probabilistic method for combinatorics in detail and introduces randomized algorithms and the theory of random graphs.

  Methods covered include the second moment method, the Rödl nibble, the Lovász Local Lemma, correlation inequalities, martingale's and tight concentration, Janson's inequality, branching processes, coupling and the differential equations method for discrete random processes. Objects studied include the configuration model for random regular graphs and the phase transition in the Erdős-Rényi random graph. If time permits we will introduce hypergraph containers and/or the spread Lemma.

- **Text.** There is no official text. The following book contain a substantial proportion of the material we will cover.

  – *The Probabilistic Method*, N. Alon and J. Spencer.

- **Prerequisites.** 21-701 Discrete Mathematics or (21-301 Combinatorics and 21-325 Probability and 21-355 Principles of Real Analysis I)

- **Web page.** `http://www.math.cmu.edu/~tbohman/21-737/probcomb.html`

- **Office hours.** Wednesday 5:00-6:00 or by arrangement. I can be reached by e-mail at `tbohman@math.cmu.edu`.

- **Homeworks.** Six homework assignments will be given during the semester. Homeworks will be due at the beginning of class on Fridays. Discussion of the homeworks is permitted, but collaboration on the writing of the assignments is not (i.e. you are NOT permitted to see the actual pages another student is handing in).

- **Exams.** There will be a closed-book in-class midterm on October 24 and a comprehensive final exam.

- **Final Exam or Paper Presentations.** Graduate students who have already passed the ACO qualifying exam or math basic exams may opt out of the final exam by presenting a paper to the class. These presentations will be 25 minutes long. (These presentations will be scheduled outside of normal class times if a large number of students take this option. Students who take this option will be asked to attend the paper presentations that occur outside class times.)

- **Accommodations for students with disabilities.** If you have a disability and have an accommodations letter from the Disability Resources office, I encourage you to discuss your accommodations and needs with me as early in the semester as possible. I will work with you to ensure that accommodations are provided as appropriate. If you suspect that you may have a disability and would benefit from accommodations but are not yet registered with the Office of Disability Resources, I encourage you to contact them at access@andrew.cmu.edu.

- **Wellness.** As a student, you may experience a range of challenges that can interfere with learning, such as strained relationships, increased anxiety, substance use, feeling down, difficulty concentrating and/or lack of motivation. These mental health concerns or stressful events may diminish your academic performance and/or reduce your ability to participate in daily activities. CMU services are available, and treatment does work. You can learn more about confidential mental health services available on campus at: http://www.cmu.edu/counseling/. Support is always available (24/7) from Counseling and Psychological Services: 412-268-2922.
- **Identity.** All people have the right to be addressed and referred to in accordance with their personal identity. I will do my best to address and refer to all students accordingly and support classmates in doing so as well. I suggest that students use their Zoom profile to express preferences in this regard but should also contact me if any issues arise.
- **Learning Objectives.** Upon completion of this course students should have command of the central objects, methods and ideas that comprise probabilistic combinatorics. This includes an understanding of probabilistic existence proofs, the second moment method, the Poisson paradigm, the Lovasz Local Lemma, threshold and phase transitions, and martingale methods.