21-737 Probabilistic Combinatorics
Homework IV: Hints
Follow-up solutions due Friday, May 12

1. Follow the argument from lecture, using breadth first search. When the search adds a vertex u (to the component in $G + F$ containing v) also add the full component in G that contains u. Couple this process with a branching process as in lecture.

   You will have to include an argument that shows that at most $n - \Omega(n)$ vertices v have 'small' components in $G + F$. Use the second moment.

2. We may assume $y \neq x$. Consider a chain $X(0), X(1), X(2), \ldots$ where $X(0) = x$. One of the sums is the expected number of visits to y in the first n steps of this chain. Get another expression for this by considering the hitting time T, which we define to be the time of the first visit to y. (And note that $P^0_{y,y} = 1$.)

3. Consider the the k gaps between tokens. At time T every gap has either disappeared or grown to length n. And the length of each gap changes when we move a token (or collection of tokens) on the boundary of the gap. Use linearity of expectations.

4. Bound $Pr(\tau_{couple} \leq jt_0)$ for all positive integers j.

5. Apply Hoeffding-Azuma.