1. Follow the argument from lecture, using breadth first search. When the search adds a vertex \( u \) (to the component in \( G + F \) containing \( v \)) also add the full component in \( G \) that contains \( u \). Couple this process with a branching process as in lecture.

You will have to include an argument that shows that at most \( n - \Omega(n) \) vertices \( v \) have ‘small’ components in \( G + F \). Use the second moment.

2. We may assume \( y \neq x \). Consider a chain \( X(0), X(1), X(2), \ldots \) where \( X(0) = x \). One of the sums is the expected number of visits to \( y \) in the first \( n \) steps of this chain. Get another expression for this by considering the hitting time \( T \), which we define to be the time of the first visit to \( y \). (And note that \( P_{y,y}^0 = 1 \).)

3. Consider the the \( k \) gaps between tokens. At time \( T \) every gap has either disappeared or grown to length \( n \). And the length of each gap changes when we move a token (or collection of tokens) on the boundary of the gap. Use linearity of expectations.

4. Bound \( Pr(\tau_{\text{couple}} \leq jt_0) \) for all positive integers \( j \).

5. Apply Hoeffding-Azuma.