21-737 Probabilistic Combinatorics
Homework III: Hints
Follow-up solutions due Friday, March 10

1. The threshold is $n^{-2/3}$. For $p \ll n^{-2/3}$ a very simple first moment calculation shows that no such collection of triangles appears. For $p \gg n^{-2/3}$ show that every set of $n/2$ vertices contains a triangle. (This actually also works for some $p = \Omega(n^{-2/3})$).

2. Start by proving the following:

   If $A_1, A_2, \ldots, A_n$ are a collection of subsets of a set Ω then
   $$|\{x \in \Omega : \exists! i \text{ such that } x \in A_i\}| = \sum_{I \subseteq [n]} |I|(-1)^{|I|-1}|A_I|$$
   where $A_I = \cap_{i \in I} A_i$.

   Prove this by reversing a sum.

3. The statement is false. Of course, in order to construct a counterexample we want to consider events with small covariance but a large intersection...

4. (a) Use Bonferroni.
   (b) Simply count the number of configurations that project onto a given simple graph.
   (c) Use question 2.