

21-737 Probabilistic Combinatorics
Homework I: Hints
Follow-up solutions due Friday, February 10

1. (b) Consider isolated vertices.
2. Let $|\mathcal{H}| = f \cdot 2^k$ where f is some function of k . Let the size of the vertex set be $2n$ and consider a random set X of $(1 + \epsilon)n$ vertices, where ϵ is also a function of k .

Carefully approximate the expected number of edges in X . *It may be useful to use the estimate $0 < b < a$ implies $\frac{1-a}{1-b} < 1 - (a - b)$.*

3. Consider a collection of random maps from the graph that contains no copy of H to the complete graph K_n .
4. Combine Chebyshev's inequality with pigeonhole. For an example of a similar argument, see the application of the second moment to the distinct subset sums problem in Section 4.6 of Alon and Spencer.

5. Note that we have

$$\mu^\ell = \left(\sum_{i=1}^m Pr(A_i) \right)^\ell.$$

What happens when we expand the right hand side?