21-228 Discrete Mathematics
Assignment # 8
Due: Friday, December 5

1. Prove that a connected planar bipartite graph with $n \geq 4$ vertices has at most $2n - 4$ edges.

2. Let $V = \{1, 2, \dots, n\}$ and let $G = (V, E)$ be a tree. Let $A_1, A_2, \dots, A_n$ be subsets of a finite set Ω. Prove that
$$\left| \bigcup_{i=1}^{n} A_i \right| \leq \sum_{i=1}^{n} |A_i| - \sum_{\{u,v\} \in E} |A_u \cap A_v| .$$

3. Suppose $G = (V, E)$ is a 3-regular connected plane graph in which the boundary of every face is either a hexagon (i.e. a cycle of length 6) or a pentagon (i.e. a cycle of length 5). Show that G has exactly 12 pentagonal faces.