

21-228 Discrete Mathematics

Assignment # 8

Due: Friday, April 25

1. Solve the recurrence

$$h_n = 5h_{n-1} - 6h_{n-2} - 4h_{n-3} + 8h_{n-4} \quad \text{for } n \geq 4$$

with initial conditions $h_0 = 0, h_1 = 1, h_2 = 1,$ and $h_3 = 15.$

2. Let $2n$ equally spaced points be placed on a circle. Let b_n denote the number of ways to join these points in pairs so that the resulting line segments do not intersect. Write b_n as a function of $b_0, b_1, b_2, \dots, b_{n-1}.$
3. Compute the sum $\sum_{i=0}^n (-1)^i \binom{n}{i} \binom{n}{n-i}.$
4. Let a_0, a_1, a_2, \dots be the sequence defined by the recurrence relation

$$a_n = a_{n-1} + 2a_{n-2} + 1 \quad \text{for } n \geq 2$$

with initial conditions $a_0 = 0$ and $a_1 = 1.$ Determine the generating function for this sequence, and use the generating function to determine a_n for all $n.$

5. Determine the generating function for the number h_n of bags of fruit of apples, oranges, bananas and pears in which there are:
- an even number of apples,
 - at most two oranges,
 - a multiple of three bananas, and
 - at most one pear.

Then use the generating function to find a formula for $h_n.$