21-228 Discrete Mathematics
Assignment # 8
Due: Friday, April 26

1. A connected graph $G = (V, E)$ has $|V| = 2k + 1$ vertices and exactly $k + 1$ vertices of degree 2, no two of which are adjacent. Show that G is not Hamiltonian.

2. Let T be a tree with n vertices, $n \geq 2$. For each positive integer i let p_i be the number of vertices of T of degree i. Prove
$$p_1 - p_3 - 2p_4 - \cdots - (n-3)p_{n-1} = 2.$$

3. Let $V = \{1, 2, \ldots, n\}$ and let $G = (V, E)$ be a tree. Let $A_1, A_2, \ldots, A_n$ be subsets of a finite set Ω. Prove that
$$\left|\bigcup_{i=1}^{n} A_i\right| \leq \sum_{i=1}^{n} |A_i| - \sum_{\{u,v\} \in E} |A_u \cap A_v|.$$

4. Suppose $G = (V, E)$ is a 3-regular connected plane graph in which the boundary of every face is either a hexagon (i.e. a cycle of length 6) or a pentagon (i.e. a cycle of length 5). Prove that G has exactly 12 pentagonal faces.