1. Adapt the proof of Inclusion/Exclusion given in class to prove the follows version of the Principle of Inclusion/Exclusion.

If $A_1, \ldots, A_n$ are events in a probability space defined on the set $\Omega$ then

$$
Pr \left( \Omega \setminus \left( \bigcup_{i=1}^{n} A_i \right) \right) = \sum_{I \subseteq \{1, \ldots, n\}} (-1)^{|I|} Pr(A_I)
$$

where

$$
A_I = \begin{cases} 
\bigcap_{i \in I} A_i & \text{if } I \neq \emptyset \\
\Omega & \text{if } I = \emptyset.
\end{cases}
$$

2. A collection of $n$ teams plays a round robin tournament. (I.e. a tournament in which every team plays every other team exactly once. There will be $\binom{n}{2}$ games in this tournament). We say that the tournament has property $S_k$ if for every set $S$ of $k$ teams there is some team that beats all teams in the set $S$. For example if we have 3 teams then the tournament would have property $S_1$ if, for example, team 1 beats team 2, team 2 beats team 3 and team 3 beats team 1. Prove that if

$$
\binom{n}{k} (1 - 2^{-k})^{n-k} < 1
$$

then it is possible that a tournament with $n$ teams has property $S_k$.

3. Suppose we choose a function $f: \{1, 2, \ldots, n\} \to \{1, 2, \ldots, 3n\}$ uniformly at random. Let the random variable $X$ be the number of elements $i$ of the domain such that $f(i) = i$. Determine $E[X]$.

4. Suppose we color the edge set of $K_n$ with 2 colors uniformly at random (so, we have $|\Omega| = 2^{\binom{n}{2}}$ and the uniform distribution). Let $X$ be the number of monochromatic triangles in the random coloring. Determine $E(X)$. 