1. Suppose 6n fair dice are rolled. Define a probability space that describes this experiment. Let \( p_n \) be the probability that every number (i.e. the numbers 1 through 6) appears exactly \( n \) times in one roll of the collection of dice. Give an exact expression for \( p_n \). Find a simpler function \( f_n \) such that \( p_n \sim f_n \).

2. A women walks randomly on the \( n \times n \) grid \( \{(x, y) : x, y \in \{1, \ldots, n\}\} \) starting at the point \((1, 1)\) (i.e. the lower left corner). Each minute the women moves either to the right or up (i.e. a move of the form \((a, b) \rightarrow (a+1, b)\) or a move of the form \((a, b) \rightarrow (a, b+1)\)). Her walk ends when she reaches the upper right corner, the point \((n, n)\). At each stage in which the woman has a choice of 2 moves she flips a fair coin to determine her next move. (If the woman is on the right edge (i.e. \((x, y) \) such that \(x = n\)) she automatically moves up and if she is on the top edge (i.e. \((x, y) \) such that \(y = n\)) she automatically moves right.) Define a probability space that describes this random walk. What is the probability that the woman reaches the top row of the grid before reaching \((n, n)\)? Explain your answer.

We say that a pair of events \( A, B \) in a probability space are independent if
\[
P(A \cap B) = P(A)P(B).\]
A collection of events \( A_1, A_2, \ldots, A_n \) is mutually independent (or jointly independent) if
\[
P\left(\bigcap_{i \in I} A_i\right) = \prod_{i \in I} P(A_i) \quad \text{for all nonempty } I \subseteq \{1, 2, \ldots, n\}.
\]

3. Let \( A \) and \( B \) be independent events in a probability space defined on the set \( \Omega \). Prove that \( \overline{A} = \Omega \setminus A \) and \( \overline{B} = \Omega \setminus B \) are independent events.

4. Define a probability space with three events \( A, B, C \) with the following properties:
   (a) \( A \) and \( B \) are independent events,
   (b) \( A \) and \( C \) are independent events,
   (c) \( B \) and \( C \) are independent events, but
   (d) \( A, B, C \) is not a mutually independent collection of events.