21-228 Discrete Mathematics
Assignment # 5
Due: Friday, March 15

1. Let the sequence $a_0, a_1, \ldots$ be defined by $a_0 = 2, a_1 = 8$ and $a_i = \sqrt{a_{i-1}a_{i-2}}$ for $i \geq 2$. Determine $\lim_{n\to\infty} a_n$.

   Hint: This is a generating functions question.

2. Prove that any edge coloring of the edge set of K_{17} with the colors Red, Blue and Green has a monochromatic triangle.

3. Let $k \geq 3$ and $n = (k-1)^2$. Give an explicit 2-coloring of the edges of K_n that does not have a monochromatic K_k.

4. Prove $R(3,5) > 11$.

   Hint: Modify the argument that we used to show $R(3,4) > 8$.

5. We say that a pair of events A, B in a probablity space are **independent** if

$$\mathbb{P}(A \cap B) = \mathbb{P}(A)\mathbb{P}(B).$$

   (a) Let A and B be independent events in a probability space defined on the set Ω. Prove that $\overline{A} = \Omega \setminus A$ and $\overline{B} = \Omega \setminus B$ are independent events.

   (b) Define a probability space with three events A, B, C with the following properties:

      i. A and B are independent events,

      ii. A and C are independent events,

      iii. B and C are independent events, but

      iv. $\mathbb{P}(A \cap B \cap C) \neq \mathbb{P}(A)\mathbb{P}(B)\mathbb{P}(C)$.

6. A women walks randomly on the $n \times n$ grid $\{(x,y) : x, y \in \{1, \ldots, n\}\}$ starting at the point $(1,1)$ (i.e. the lower left corner). Each minute the women moves either to the right or up (i.e. a move of the form $(a,b) \to (a+1,b)$ or a move of the form $(a,b) \to (a,b+1)$). Her walk ends when she reaches the upper right corner, the point (n,n). At each stage in which the woman has a choice of 2 moves she flips a fair coin to determine her next move. (If the woman is on the right edge (i.e. (x,y) such that $x = n$) she automatically moves up and if she is on the top edge (i.e. (x,y) such that $y = n$) she automatically moves right.) Define a probability space that describes this random walk. What is the probability that the woman reaches the top row of the grid before reaching (n,n)? Explain your answer.