1. Let the sequence \( a_0, a_1, \ldots \) be defined by \( a_0 = 2, a_1 = 8 \) and \( a_i = \sqrt{a_{i-1}a_{i-2}} \) for \( i \geq 2 \). Determine \( \lim_{n \to \infty} a_n \).

2. Prove that any edge coloring of the edge set of \( K_{17} \) with the colors Red, Blue and Green has a monochromatic triangle.

3. Let \( k \geq 3 \) and \( n = (k - 1)^2 \). Give an explicit 2-coloring of the edges of \( K_n \) that does not have a monochromatic \( K_k \).

4. Prove \( R(3, 5) \leq 14 \).

5. If \( G = (V, E) \) is a graph and \( v \in V \) then the degree of \( v \), denoted \( d(v) \), is the number of edges in \( G \) that contain \( v \) (e.g. the degree of every vertex in the complete graph \( K_n \) is \( n - 1 \)).

   Let \( n \geq 2 \) be an integer. Does there exist a graph with vertex set \( V = \{v_1, \ldots, v_n\} \) such that \( d(v_i) = i - 1 \) for \( i = 1, \ldots, n \)?

6. A graph \( G = (V, E) \) is bipartite if there exists a partition \( V = A \cup B \) such that

   \[
   E \cap \binom{A}{2} = \emptyset \quad \text{and} \quad E \cap \binom{B}{2} = \emptyset.
   \]

   In other words, every edge has one vertex in \( A \) and one vertex in \( B \). The sets \( A \) and \( B \) are the parts of the bipartition of \( G \).

   A graph \( G = (V, E) \) is \( d \)-regular if every vertex in \( G \) has degree \( d \).

   Let \( G \) be a \( d \)-regular bipartite graph with parts \( A, B \). Prove that if \( d \geq 1 \) then \( |A| = |B| \).