21-228 Discrete Mathematics
Assignment # 4
Due: Friday, February 22

1. Prove that of any five points chosen within an equilateral triangle with side length 1 there are two whose distance apart is at most $1/2$.

2. Prove that any edge coloring of the edge set of K_{17} with the colors Red, Blue and Green has a monochromatic triangle.

3. If $G = (V, E)$ is a graph and $v \in V$ then the **degree** of v, denoted $d(v)$, is the number of edges in G that contain v (e.g. the degree of every vertex in the complete graph K_n is $n - 1$).

   (a) Does there exist a graph with vertex set $V = \{v_1, \dots, v_n\}$ such that $d(v_i) = i - 1$ for $i = 1, \dots, n$?

   (b) Does there exist a graph with vertex set $V = \{v_1, v_2, \dots, v_7\}$ such that $d(v_i) = 3$ for $i = 1, \dots, 7$?

4. Suppose $6n$ fair dice are rolled. Define a probability space that describes this experiment. What is the probability that every number (i.e. the numbers 1 through 6) appears exactly n times in one roll of the collection of dice?

5. A women walks randomly on the $n \times n$ grid $\{(x, y) : x, y \in \{1, \dots, n\}\}$ starting at the point $(1, 1)$ (i.e. the lower left corner). Each minute the women moves either to the right or up (i.e. a move of the form $(a, b) \to (a + 1, b)$ or a move of the form $(a, b) \to (a, b + 1)$). Her walk ends when she reaches the upper right corner, the point (n, n). At each stage in which the woman has a choice of 2 moves she flips a fair coin to determine her next move. (If the woman is on the right edge (i.e. (x, y) such that $x = n$) she automatically moves up and if she is on the top edge (i.e. (x, y) such that $y = n$) she automatically moves right.) Define a probability space that describes this random walk. What is the probability that the woman reaches the top row of the grid before reaching (n, n)? Explain your answer.