21-228 Discrete Mathematics
Assignment # 2
Due: Friday, February 1

1. There are 4 candidates in an election. A population of m people cast ballots (each person votes for exactly one of the 4 candidates). What is the number of possible vote totals in the election? (E.g. if $m = 2$ then there are 10 possible outcomes.)

2. Prove that for $n \in \mathbb{P}$ we have
$$2\sqrt{n+1} - 2 < 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{n}} \leq 2\sqrt{n} - 1.$$

3. Prove the following bound:
$$n! \leq e\sqrt{n}\left(\frac{n}{e}\right)^n.$$
Hint. Consider the lower bound on $n!$ *given in lecture, but from the area below the curve* $y = \log x$ *subtract the areas of suitable triangles.*

4. Find the number of integers between 1 and 10,000 that are neither prefect squares nor perfect cubes.

5. Use Inclusion/Exclusion to determine the number of functions
$$f : \{1, 2, \ldots, 9\} \to \{1, 2, \ldots, 9\}$$
such that $f(i) = i$ for at least one odd number.

6. A bag contains 100 apples, 100 bananas, 100 oranges, and 100 pears. I pick n pieces of fruit out of the bag. How large must n to assure that my collection of n pieces of fruit contains a dozen pieces of the same kind?