21-228 Discrete Mathematics Assignment # 1

Due: Friday, September 5

1. Find all positive integers a > b > c for which

$$\binom{a}{b} \binom{b}{c} = 2 \binom{a}{c}.$$

Give a combinatorial justification for your answer (i.e. appeal to the fact that $\binom{a}{b}$ gives the number of subsets of an a-element set having exactly b elements).

- 2. If m indistinguishable 4-sided die are rolled how many distinguishable outcomes are there? (E.g. if m=2 then there are 10 possibilities.) How many outcomes are there in which each of the four numbers appear at least once?
- 3. Let p and q be positive integers. How many sequences of p 1's and q 0's are there with the property that there are at least two 0's between every pair of 1's?
- 4. A function $f: \{1, 2, ..., n\} \to \{1, 2, ..., n\}$ is **monotone** if i < j implies $f(i) \le f(j)$. How many monotone functions $f: \{1, 2, ..., n\} \to \{1, 2, ..., n\}$ are there?
- 5. Let T be a set such that |T| = n. How many sequences $S_1 \subseteq S_2 \subseteq \cdots \subseteq S_k$ such that $S_i \subseteq T$ for $i = 1, \ldots, k$ are there?
- 6. Use a **combinatorial** argument to prove the following: for all positive integers m_1, m_2, n we have

$$\sum_{k=0}^{n} {m_1 \choose k} {m_2 \choose n-k} = {m_1+m_2 \choose n}.$$