21-228 Discrete Mathematics
Assignment # 1
Due: Friday, January 25

1. Let $X = \{1, \ldots, n\}$. Set
$$\mathcal{F}_1 = \{A \subseteq X : 1 \in A\},$$
and for $i = 2, \ldots, n$ define
$$\mathcal{F}_i = \{A \subseteq [n] : i \in A \text{ and } 1, \ldots, i-1 \notin A\}.$$
Does the collection $\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_n$ form a partition of 2^X? If it is not a partition, explain why not. If it is a partition give the equivalence relation that defines the partition.

2. What is the number of ways to color n indistinguishable objects with 3 colors?

3. What is the number of ways to color n indistinguishable objects with 3 colors if each color must be used at least once?

4. Find all positive integers $a > b > c$ for which
$$\binom{a}{b}\binom{b}{c} = 2\binom{a}{c}.$$
Give a combinatorial justification for your answer (i.e. appeal to the fact that $\binom{a}{b}$ gives the number of subsets of an a-element set having exactly b elements).

5. Give a combinatorial (i.e. bijective) proof of the following identity:
$$n2^{n-1} = \sum_{i=1}^{n} i\binom{n}{i}.$$