The Mathematical Structure of Economic Theory

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Abstract: We will be investigating some basic questions economic theory: do individuals behave as the theory predicts, that is, do they maximize some utility function? If so, can their preferences be inferred from observed behaviour? We will show this is the case for individuals, and then we will turn to couples and finally to markets. Answering these questions requires mathematical tools which were developed by Elie Cartan and others for the purpose of studying systems of partial differential equations arising in geometry, but which now turn out to be crucial in other settings. We will spend about half the time explaining these methods, and half the time applying them to economic situations

Lecture 1: Motivation: characterization, identification, the unitary model, the Slutsky relations

Lecture 2: Differential forms, exterior products and derivatives, theorems of Frobenius, Darboux, and Ekeland-Nirenberg

Lecture 3: Households, the efficiency assumption, the Browning-Chiappori condition, characterization and identification

Lecture 4: Exterior differential systems, integral manifolds, the Cartan-Kähler theorem

Lecture 5: Markets with many agents: excess demand (Sonnenschein-Mantel-Debreu theorem), market demand (Chiappori-Ekeland theorem)