MATH 820 : ADVANCED TOPICS IN ANALYSIS
VARIATIONAL AND PDE TECHNIQUES IN DATA ANALYSIS
Spring 2016

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Lectures MWF 9:30 in Wean Hall 7218
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Learning Objectives. The objective of the course it to learn about applications of variational and PDE techniques in data analysis and image processing. This includes learning about tasks of data analysis such as clustering, classification, semi-supervised learning dimensional reduction, and image processing tasks such as image comparison and registration. In the process the mathematical techniques needed to understand these problems will be introduced. In particular you will learn techniques of optimal transportation and calculus of variations. You will explore how these techniques are used in modeling, analysis and computation.

Prerequisites. knowledge of Measure and Integration and Functional Analysis, familiarity with calculus of variations PDE, Differential Geometry, Sobolev Spaces, and Probability.

Evaluation. The course grade is based on final project. The final project involves writing a document of at least 6 pages and giving a 25 minute oral presentation. There are two options for the final project:

1. *Explore a research idea and report the results.* There are truly many questions in data analysis that can be formulated in terms of variational and PDE problems that have not been explored. Your task would be to (try to) explore one of them. This can include modeling (that is giving the particular data analysis task a variational or PDE description), computational experiments (to learn how the model works), and analysis (establishing the mathematical properties of objects, for example proving that the variational problem has a unique C^1 solution).

2. *Read published research on a topic and explain its main ideas.* Typically this would constitute of reading a paper on a topic and explaining it to your peers in both written form (the report) and as an oral presentation. Reading on the subject can include getting familiar with the background material and the exploring the related publications on the topic.

Approximate Outline [1]

I. Optimal transportation and related metrics

- Optimal transportation (OT maps, OT plans, existence, Kantorovich duality, existence of transportation potentials, Brenier's theorem, generalization to other costs, Monge-Ampere equation, geometry of optimal transportation, Benamou-Brenier theorem, Numerical approximation of OT)
- Other transportation based distances (unbalanced transport, Hellinger–Kantorovich, Wasserstein-Fisher-Rao, TL^p, entropy regularized optimal transport)
- deformation based morphometry (image registration, large deformation diffeomorphic matching)

II. Calculus of variations and problems involving perimeter

- Brief review of the spectral theory of the Laplace operator
- Total variation, BV functions
- Perimeter, minimal surfaces
- Cheeger inequality
- Γ-convergence (definition, nonlocal TV, convergence of nonlocal TV to TV)
- Motion by mean curvature

III. Optimization on graphs and clustering on graphs

- Graph Laplacian and spectral clustering
- Total variation on graphs
- Minimal cut, Cheeger cut
- Γ-convergence to continuum problems as number of available data increases
- Optimal transportation on graphs (Erbar, Maas and others)

IV. Data parameterization and low-dimensional embeddings

- Principal component analysis, multidimensional scaling
- Principal curves, Average distance problem
- laplacian eigemnaps, diffusion maps

V. Comparing sets of points

- Gromov–Hausdorff–Wasserstein distance (Memoli)
- Functional matching (Ovsjanikov et al.)

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[1] The topics may change depending on interest etc.