

MATH 720 : MEASURE AND INTEGRATION

Fall 2018

Instructor: Dejan Slepčev

Lectures MWF 2:30 in Wean Hall 7218

Office: 7123 Wean Hall

Office Hours: Monday 11:00-12:00, Wednesday 3:30-4:30 and by appointment

Phone: 268-2562

Email: slepcev@math.cmu.edu

Text: *Real Analysis (2nd edition)* by Gerald Folland, 1999 Wiley.

Learning Objectives. You will learn some of the fundamental objects of real analysis. The notions of measure and integral developed are essential to analysis, probability, and statistics. The notion of measure generalizes the notion of volume introduced in calculus and the notion of Lebesgue integral generalizes the notion of the Riemann integral and overcomes its limitations. You will develop a working knowledge of measures and integrals, as well as of the related functional spaces.

Prerequisites. Knowledge of real analysis (at the level of an undergraduate course such as Math 355 and 356). This includes basic knowledge of metric spaces.

Evaluation. The course grade will be based on problem sets (50%), a midterm exam (20%), and a final exam (30%). Midterm exam will take place on Friday, October 5th. The exams are closed book exams, however everyone will be allowed to bring one (two sided) letter sized sheet of notes.

Problem sets. There will be about 8 problem sets. The problem sets and due dates will be posted on the course's Canvas page. Late homework will not receive score. However, if you have a valid reason for not doing a problem set (illness for example), the particular homework will not count towards your grade. Discussing the problem sets with your classmates is fine, as long as you are only exchanging ideas and general knowledge, and not the solutions to the problems. In particular everyone should present his/her own solutions.

Few general remarks from the administration. Take care of yourself. Do your best to maintain a healthy lifestyle this semester by eating well, exercising, avoiding drugs and alcohol, getting enough sleep and taking some time to relax. This will help you achieve your goals and cope with stress.

All of us benefit from support during times of struggle. You are not alone. There are many helpful resources available on campus and an important part of the college experience is learning how to ask for help. Asking for support sooner rather than later is often helpful. If you or anyone you know experiences any academic stress, difficult life events, or feelings like anxiety or depression, we strongly encourage you to seek support. Counseling and Psychological Services (CaPS) is here to help: call 412-268-2922 and visit their website at <http://www.cmu.edu/counseling/>. Consider reaching out to a friend, faculty or family member you trust for help getting connected to the support that can help.

If you or someone you know is feeling suicidal or in danger of self-harm, call someone immediately, day or night: CaPS (412-268-2922) or Re:solve Crisis Network (888-796-8226). If the situation is life threatening, call the police. CMU Police (412-268-2323).

APPROXIMATE OUTLINE

I. Measures

1. σ -algebras, measures, construction of measures (outer measures, Carathéodory Extension Theorem), Borel measures, Lebesgue measure in 1D

II. Integration

1. measurable functions, integration of nonnegative functions (Monotone Convergence Theorem, Fatou Lemma), integration of complex functions (Dominated Convergence Theorem), modes of convergence (Egoroff Theorem, Lusin Theorem, equi-integrability, Vitali Convergence Theorem)
2. product measures (Monotone Class Theorem, Fubini Theorem), n-dimensional Lebesgue measure, Lebesgue integral,

III. Signed Measures and Differentiation

1. signed measures (Hahn decomposition, Jordan decomposition), Radon-Nikodym derivative, Lebesgue Decomposition Theorem, Hardy-Littlewood maximal function, Vitali Covering Theorem, Lebesgue Differentiation Theorem, change of variables.

IV. L^p Spaces

1. Normed spaces, Banach spaces, duality, Hilbert spaces
2. L^p -spaces, Hölder and Minkowski inequalities, dual of L^p

V. Radon Measures (*time permitting)